Solutions to the exercises on Finite Automata

April, 2007

Exercise on slide 4

Given a state diagram of FA $M1$. Is 000111 accepted, is 10110?

Solution

000111 and 10110 are accepted.

Exercises on slide 7

Exercise 1

What is the language recognized by $M1$?

Solution

The language recognized by $M1$ is the set of all strings that contain 11 as a substring.

Exercise 2

1. Define a recognizer for $\{w \in \{0,1\}^* : w$ contains at least four 1's$\}$.
2. Is there a recognizer for $\{w \in \{0,1\}^* : w$ contains the substring $1010\}$?

Solution

Such recognizers are on Figure 3 for task 1 and on Figure 4 for task 2.

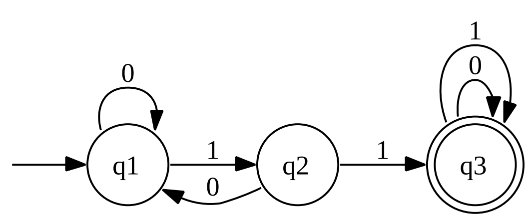

Figure 1. $M1$

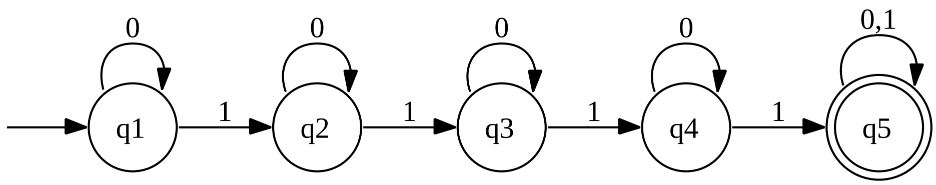

Figure 3. Recognizer for $\{w \in \{0,1\}^* : w$ contains at least four 1's$\}$

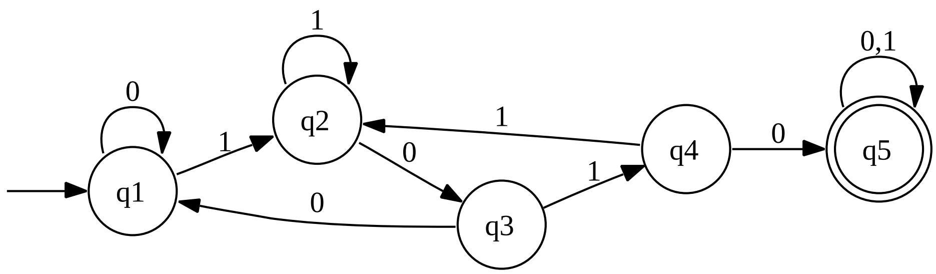

Figure 4. Recognizer for $\{w \in \{0,1\}^* : w$ contains the substring $1010\}$

Exercise 3

Let $\Sigma = \{0, 1\}$. Define a recognizer for $\emptyset$.

Solution

Such recognizer is on Figure 5.

Exercise 4

Let $\Sigma = \{0, 1\}$. Define a recognizer for $\{\varepsilon\}$.

Solution

Such recognizer is on Figure 6.

Exercise on slide 11

What is the language recognized by $N1$ (Fig.7) and by $N2$ (Fig.8)?

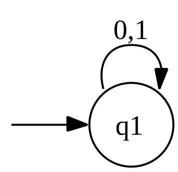

Figure 5. Recognizer for $\emptyset$

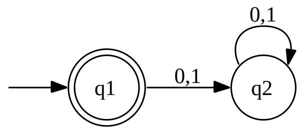

Figure 6. Recognizer for $\{\varepsilon\}$

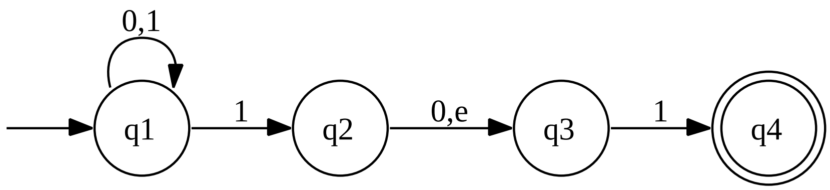

Figure 7. $N1$

Solution

$L(N1)$ is the set of strings over $\{0, 1\}$ ending on 11 or 101
$L(N2) =$ is the set of strings over $\{0, 1\}$ containing 101

Exercise on slide 13

Show a sequence of states in $N2$ that makes the string 1011 be accepted.

Solution

$q_1, q_2, q_3, q_4, q_5, q_5$, where: $q_2 \in \delta(q_1, 1) = \{q_1, q_2\}$
$q_3 \in \delta(q_2, 0) = \{q_3\}$
$q_4 \in \delta(q_3, 1) = \{q_4\}$
$q_5 \in \delta(q_4, \varepsilon) = \{q_5\}$
$q_5 \in \delta(q_5, 1) = \{q_5\}$
Concurrent computation steps are: $\{q_1, 1011\} \rightarrow \{(q_1, 011), (q_2, 011)\} \rightarrow \{(q_1, 11), (q_3, 11)\} \rightarrow \{(q_1, 1), (q_2, 1), (q_4, 1)\} \rightarrow \{(q_1, \varepsilon), (q_2, \varepsilon), (q_2, 1), (q_5, 1)\} \rightarrow \{(q_1, \varepsilon), (q_2, \varepsilon), (q_2, 1), (q_5, \varepsilon)\}$

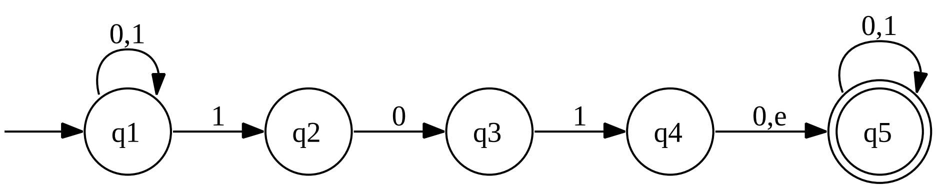

Figure 8. $N2$

Exercises on pages 653-655 in Discrete Mathematics and Its Applications

Exercise 3

Find all pairs of sets of strings A and B for which $AB = \{10, 111, 1010, 1000, 10111, 101000\}$.

Solution

a) $A = \emptyset$ and $B = \{10, 111, 1010, 1000, 10111, 101000\}$
b) $B = \emptyset$ and $A = \{10, 111, 1010, 1000, 10111, 101000\}$
c) $A = \{1\}$ and $B = \{0, 11, 010, 000, 0111, 01000\}$
d) $A = \{1, 101\}$ and $B = \{0, 11, 000\}$
e) $A = \{\varepsilon, 10\}$ and $B = \{10, 111, 1000\}$

Exercises 12-16

In exercises 12-16 find the languages recognized by the given deterministic finite-state automaton.

Solutions

12) $\varepsilon \cup ((1 \cup 01^*0)(0 \cup 1)^*)$
13) $0 \cup (1(0 \cup 1))(0 \cup 1)^*$
14) $\varepsilon \cup 01^*$
15) 0^*11^*
16) $1^*00^*(1(0 \cup 1)(0 \cup 1)^*) \cup 1^*00^*$

Exercise 17,19,21

In exercises 17,19,21 find the languages recognized by the given nondeterministic finite-state automaton.

Solutions

17) $01 \cup 11 \cup 0$
19) $\varepsilon \cup ((0 \cup 00^*1)1^*)$
21) $10^* \cup 10^*10^*$

Exercise 24

Find a deterministic finite-state automaton that recognizes the same language as the nondeterministic finite-state automaton in Exercise 19.

Solution

Let such DFA be $M = (Q, \Sigma, \delta, q_0, F)$. Where $Q \subseteq \{\emptyset, \{s_0\}, \{s_1\}, \{s_2\}, \{s_0, s_1\}, \{s_0, s_2\}, \{s_1, s_2\}, \{s_0, s_1, s_2\}\}$, $\Sigma = \{0, 1\}$ and $q_0 = \{s_0\}$. Let us make the transition function δ:
$\delta(q_0, 0) = \{s_1, s_2\} = q_1$

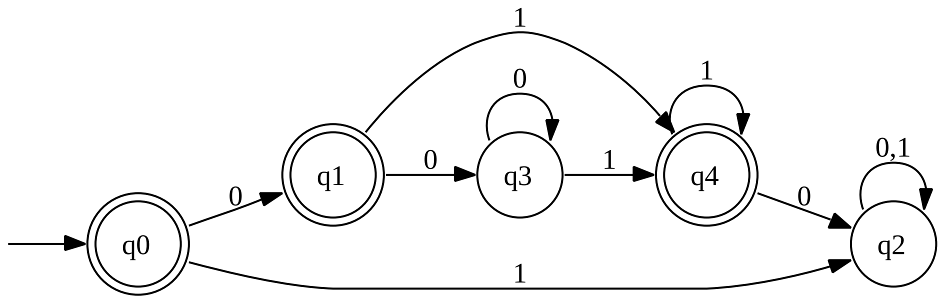

Figure 9. Deterministic finite-state automaton for Exercise 24

$\delta(q_0, 1) = \emptyset = q_2$
$\delta(q_1, 0) = \{s_1\} = q_3$
$\delta(q_1, 1) = \{s_2\} = q_4$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = q_2$
$\delta(q_3, 0) = \{s_1\} = q_3$
$\delta(q_3, 1) = \{s_2\} = q_4$
$\delta(q_4, 0) = \emptyset = q_2$
$\delta(q_4, 1) = \{s_2\} = q_4$.
As s_0 and s_2 are accept states, $F = \{q_0, q_1, q_4\}$. The required deterministic finite-state automaton is represented on Figure 9.

Exercise 26

Find a deterministic finite-state automaton that recognizes the same language as the nondeterministic finite-state automaton in Exercise 21.

Solution

Let such DFA be $M = (Q, \Sigma, \delta, q_0, F)$. Where $Q \subseteq \{\emptyset, \{s_0\}, \{s_1\}, \{s_2\}, \{s_3\}, \{s_0, s_1\},$
$\{s_0, s_2\}, \{s_0, s_3\}, \{s_1, s_2\}, \{s_1, s_3\}, \{s_3, s_2\}, \{s_0, s_1, s_2\},$
$\{s_0, s_1, s_3\}, \{s_0, s_3, s_2\}, \{s_3, s_1, s_2\}, \{s_0, s_1, s_2, s_4\}\}$,
$\Sigma = \{0, 1\}$ and $q_0 = \{s_0\}$. Let us make the transition function δ:
$\delta(q_0, 1) = \{s_1\} = q_1$
$\delta(q_0, 0) = \{s_2\} = q_2$
$\delta(q_1, 0) = \{s_1, s_2\} = q_3$
$\delta(q_1, 1) = \{s_3\} = q_4$
$\delta(q_2, 0) = \emptyset$
$\delta(q_2, 1) = \emptyset = q_2$, here all transitions from q_2 go to $\emptyset$, therefore state q_2 is a state for $\emptyset$
$\delta(q_3, 0) = \{s_1, s_2\} = q_3$
$\delta(q_3, 1) = \{s_3\} = q_4$
$\delta(q_4, 0) = \{s_2, s_3\} = q_5$
$\delta(q_4, 1) = \{s_2\} = q_2$
$\delta(q_5, 0) = \{s_2, s_3\} = q_5$
$\delta(q_5, 1) = \{s_2\} = q_2$.

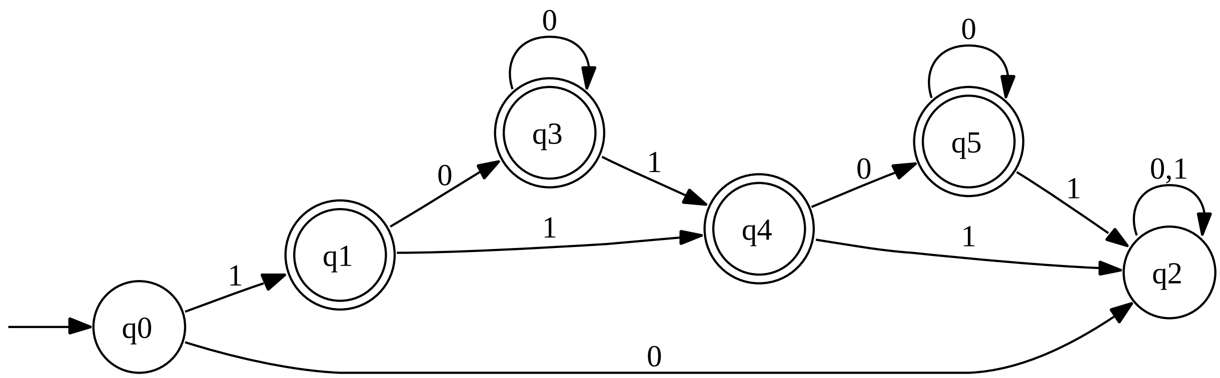

Figure 10. Deterministic finite-state automaton for Exercise 26

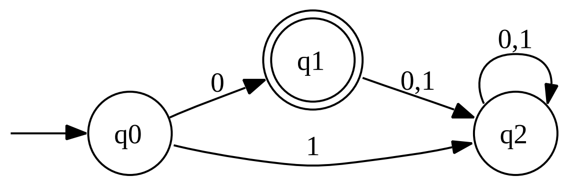

Figure 11. Deterministic finite-state automaton that recognizes $\{0\}$

As s_1 and s_3 are accept states, $F = \{q_1, q_3, q_4, q_5\}$. The required deterministic finite-state automaton is represented on Figure 10.

Exercise 27

Find a deterministic finite-state automaton that recognizes each of the following sets.
a) $\{0\}$
b) $\{1, 00\}$
c) $\{1^n | n = 2, 3, 4, ...\}$

Solution

a) Figure 11
b) Figure 12
c) Figure 13

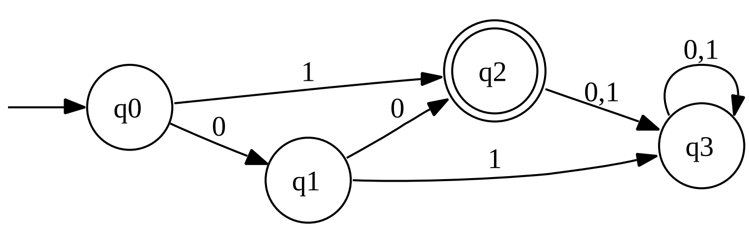

Figure 12. Deterministic finite-state automaton that recognizes $\{1, 00\}$

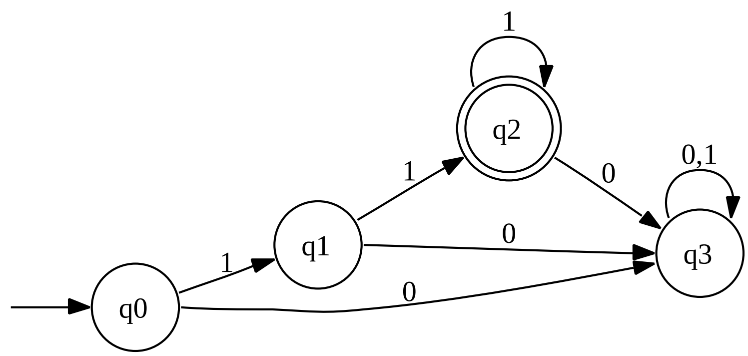

Figure 13. Deterministic finite-state automaton that recognizes $\{1^n | n = 2, 3, 4, ...\}$

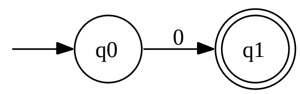

Figure 14. Nondeterministic finite-state automaton that recognizes $\{0\}$

Exercise 28

Find a nondeterministic finite-state automaton that recognizes each of the languages in Exercise 27, and has fewer states, if possible, than the deterministic automaton you found in that exercise.

Solution

a) Figure 14
b) Figure 15
c) Figure 16

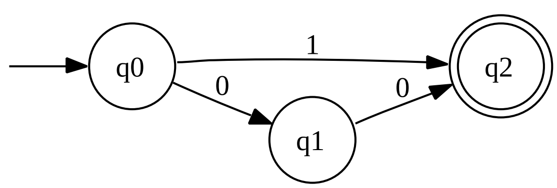

Figure 15. Nondeterministic finite-state automaton that recognizes $\{1, 00\}$

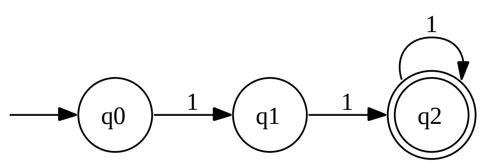

Figure 16. Nondeterministic finite-state automaton that recognizes $\{1^n | n = 2, 3, 4, ...\}$