Machine-Checked Semantic Session Typing

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Session Types - A type system for message passing

Syntax

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\begin{aligned}
S ::= \;& \mathbf{!}A.\, S && \mid \\
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$$c : \texttt{chan}\ S$$

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3. **Lack of mechanised soundness proofs for session type systems**
   - Few results exist for simpler systems
   - None exist for more expressive systems

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Semantic typing [Milner, Ahmed, Princeton PCC project, RustBelt project]

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- Modernly defined in terms of a program logic
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Key Idea

Semantic typing using **Iris** and **Actris**

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Actris [Hinrichsen et al. POPL'20]

- **Dependent separation protocols:** Logical protocols inspired by session types
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2. Supports integrating safe yet untypeable programs, through manual proofs
3. Full mechanisation in Coq (https://gitlab.mpi-sws.org/iris/actris/-/tree/cpp21)

Syntactic Typing
vs.
Semantic Typing

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  - **Progress**: if $\vdash e : A$ then $(e \in \mathsf{Val})$ or $(\exists e'.\ e \longrightarrow e')$
  - **Preservation**: if $\vdash e : A$ and $e \longrightarrow e'$ then $\vdash e' : A$

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- **Rule**
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  -

Replacing Coq's Prop with Iris's iProp implicitly threads the heap:

- similar to $\mathsf{Type} \triangleq \mathsf{Val} \to \mathsf{Heap} \to \mathsf{Prop}$
- but also handles step-indexing and user-defined ghost state

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> Iris's *weakest precondition* ($\mathsf{wp}\ e\ \{\Phi\}$):
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- R
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Replacing regular conjunction ($\wedge$) with Iris's *separation conjunction* ($*$) yields a substructural product type

The *separation conjunction* ($P * Q$) states that P and Q hold for disjoint parts of the heap

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- **Types** defined as predicates over values $\mathsf{Type} \triangleq \mathsf{Val} \to \mathsf{iProp}$

$$
\begin{aligned}
\mathsf{Z} &\triangleq \lambda\, w.\, w \in \mathbb{Z} \\
A_1 \times A_2 &\triangleq \lambda\, w.\, \exists w_1, w_2.\, (w = (w_1, w_2)) * (A_1\, w_1) * (A_2\, w_2) \\
\mathtt{ref}_{\mathtt{uniq}}\, A &\triangleq \lambda\, w.\, (w \in \mathsf{Loc}) * \exists v.\, (w \mapsto v) * (A\, v)
\end{aligned}
$$

- **Judgement** *defined* as safety-capturing evaluation

$$\vDash e : A \quad \triangleq \quad \mathsf{wp}\, e\, \{A\}$$

- **Ru**
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  - Consequence of the judgement definition

The *points-to connective* $(\ell \mapsto v)$ asserts exclusive ownership of a location ℓ, stating that it holds the value v

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A_1 \times A_2 &\triangleq \lambda\, w.\, \exists w_1, w_2.\, (w = (w_1, w_2)) * \triangleright (A_1\, w_1) * \triangleright (A_2\, w_2) \\
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Adding Iris's later modality ($\triangleright$) allows modeling equi-recursive types using Iris's guarded recursion operator ($\mu\, X.\, A$)

- **Ju**[illegible]
- **Rules** are *proven* as lemmas: $\vDash i : Z \quad \rightsquigarrow \quad i \in \mathbb{Z}$
- **Semantic type safety**: If $\vDash e : A$ then safe e
  - Consequence of the judgement definition

Semantic Typing – Typing Contexts

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$$\Gamma, (x : \texttt{chan}\ (\mathbf{!}A.\, S)), (y : A) \vdash \texttt{send}\ x\ y : 1 \dashv \Gamma, (x : \texttt{chan}\ S)$$
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Semantic judgement with contexts:

$$\boxed{\Gamma \vDash \sigma} \triangleq ?$$
$$\Gamma \vDash e : A \dashv \Gamma' \triangleq ?$$

The *closing substitution* judgement ($\Gamma \vDash \sigma$) captures separate ownership of the type predicates in context Γ for the values in closing substitution σ.

- $\Gamma \in \mathsf{List}\ (\mathsf{String} \times \mathsf{Type})$
- $\sigma \in \mathsf{String} \xrightharpoonup{\text{fin}} \mathsf{Val}$

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Semantic judgement with contexts:

$$\Gamma \vDash \sigma \triangleq \boxed{\mathop{\scalebox{2}{$\ast$}}_{(x,A)\in\Gamma.}} A(\sigma(x))$$
$$\Gamma \vDash e : A \dashv \Gamma' \triangleq ?$$

The *iterated separating conjunction* ($\mathop{\scalebox{2}{$\ast$}}$) ensures that the resources of each variable are owned separately:

$$\mathop{\scalebox{2}{$\ast$}}_{y\in y_1 \ldots y_n.}\ \Phi\, y \triangleq \Phi\, y_1 * \cdots * \Phi\, y_n$$

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$$\Gamma \vDash e : A \dashv \Gamma' \triangleq \forall \sigma.\, (\Gamma \vDash \sigma) \mathrel{-\!\!\ast} \mathrm{wp}\ e[\sigma]\ \{v.\, (A\, v) \ast (\Gamma' \vDash \sigma)\}$$

The *separating implication* ($-\!\!\ast$) is used similarly to implication as:

$$\frac{P \ast Q \vdash R}{P \vdash Q \mathrel{-\!\!\ast} R} \qquad \frac{P \land Q \vdash R}{P \vdash Q \Rightarrow R}$$

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Inspired by the RustBelt project

Semantic Term Types – Example proof

Rule:

$$\frac{\Gamma_1 \vDash e_1 : A_1 \dashv \Gamma_1' \qquad \Gamma_2 \vDash e_2 : A_2 \dashv \Gamma_2'}{\Gamma_1 \cdot \Gamma_2 \vDash (e_1 \mid\mid e_2) : (A_1 \times A_2) \dashv \Gamma_1' \cdot \Gamma_2'}$$

Proof:

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Lemma ltyped_par Γ1 Γ1' Γ2 Γ2' e1 e2 A1 A2 :
  (Γ1 ⊨ e1 : A1 ⫤ Γ1') -∗ (Γ2 ⊨ e2 : A2 ⫤ Γ2') -∗
  (Γ1 ++ Γ2 ⊨ (e1 ||| e2) : (A1 * A2) ⫤ Γ1' ++ Γ2').
Proof.
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\begin{aligned}
&(\forall \sigma_1.\, (\Gamma_1 \vDash \sigma_1) \twoheadrightarrow \mathsf{wp}\; e_1[\sigma_1]\, \{w_1.\, (A_1\, w_1) * (\Gamma_1' \vDash \sigma_1)\}) \twoheadrightarrow \\
&(\forall \sigma_2.\, (\Gamma_2 \vDash \sigma_2) \twoheadrightarrow \mathsf{wp}\; e_2[\sigma_2]\, \{w_2.\, (A_2\, w_2) * (\Gamma_2' \vDash \sigma_2)\}) \twoheadrightarrow \\
&\forall \sigma.\, (\Gamma_1 \cdot \Gamma_2 \vDash \sigma) \twoheadrightarrow \mathsf{wp}\; (e_1 || e_2)[\sigma]\; \{w.\, (\exists w_1, w_2.\, w = (w_1, w_2) * \\
&\qquad\qquad\qquad\qquad\qquad (A_1\, w_1) * (A_2\, w_2)) * \\
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"He2" : Γ2 ⊨ e2 : A2 ⫣ Γ2'
--------------------------□
"HA1" : A1 w1
"HA2" : A2 w2
--------------------------∗
(A1 × A2) (w1, w2)
```

$$A_1 \times A_2 \triangleq \lambda\, w.\, \exists w_1, w_2.\, (w = (w_1, w_2)) * \triangleright(A_1\, w_1) * \triangleright(A_2\, w_2)$$

Semantic Term Types – Example proof

Rule:

$$\frac{\Gamma_1 \vDash e_1 : A_1 \dashv \Gamma_1' \qquad \Gamma_2 \vDash e_2 : A_2 \dashv \Gamma_2'}{\Gamma_1 \cdot \Gamma_2 \vDash (e_1 \mid\mid e_2) : (A_1 \times A_2) \dashv \Gamma_1' \cdot \Gamma_2'}$$

Proof:

```
Lemma ltyped_par Γ1 Γ1' Γ2 Γ2' e1 e2 A1 A2 :
  (Γ1 ⊨ e1 : A1 ⫤ Γ1') -∗ (Γ2 ⊨ e2 : A2 ⫤ Γ2') -∗
  (Γ1 ++ Γ2 ⊨ (e1 ||| e2) : (A1 * A2) ⫤ Γ1' ++ Γ2').
Proof.
  iIntros "#He1 #He2 !>" (σ) "HΓ /=".
  iDestruct (ctx_ltyped_app with "HΓ")
    as "[HΓ1 HΓ2]".
  wp_apply (wp_par with "(He1 HΓ1) (He2 HΓ2)").
  iIntros (w1 w2) "[[HA1 HΓ1'] [HA2 HΓ2']] !>".
  iSplitL "HA1 HA2".
  + iExists w1, w2. by iFrame.
  + iApply ctx_ltyped_app. by iFrame.
Qed.
```

```
Γ1, Γ1', Γ2, Γ2' : ctx Σ
e1, e2 : expr
A1, A2 : ltty Σ
σ : gmap string val
w1, w2 : val
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Γ1' ++ Γ2' ⊨ σ
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Semantic Term Types – Example proof

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"HΓ2'" : Γ2' ⊨ σ
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Γ1' ++ Γ2' ⊨ σ
```

$$\frac{\Gamma_1 \cdot \Gamma_2 \vDash \sigma}{(\Gamma_1 \vDash \sigma) * (\Gamma_2 \vDash \sigma)}$$

Semantic Term Types – Example proof

Rule:

$$\frac{\Gamma_1 \vDash e_1 : A_1 \dashv \Gamma_1' \qquad \Gamma_2 \vDash e_2 : A_2 \dashv \Gamma_2'}{\Gamma_1 \cdot \Gamma_2 \vDash (e_1 \mid\mid e_2) : (A_1 \times A_2) \dashv \Gamma_1' \cdot \Gamma_2'}$$

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```
No more subgoals.
```

Semantic Session Type System

Language

ML-like language

$$e \in \text{Expr} ::= v \mid x \mid \texttt{rec}\, f\ x = e \mid e_1(e_2) \mid$$

Language

ML-like language extended with state

$$\begin{aligned} e \in \text{Expr} ::= {} & v \mid x \mid \texttt{rec}\, f\ x = e \mid e_1(e_2) \mid \\ & \texttt{ref}\ (e) \mid\ !\,e \mid e_1 \leftarrow e_2 \mid && \text{(state)} \end{aligned}$$

Language

ML-like language extended with state, concurrency

$$
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e \in \mathsf{Expr} ::= {}& v \mid x \mid \texttt{rec}\, f\ x = e \mid e_1(e_2) \mid \\
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$$

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& \texttt{newlock}\ () \mid \texttt{acquire}\ e \mid \texttt{release}\ e \mid && \text{(locks)}
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Language

ML-like language extended with state, concurrency, locks, and message passing

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Message-passing is:

- **Binary:** Each channel have one pair of endpoints

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Message-passing is:

- **Binary:** Each channel have one pair of endpoints
- **Asynchronous:** `send` does not block, two buffers per endpoint pair
- **Affine:** No `close` expression, channels are garbage collected

Semantic Session Types

Session types as a new type kind:

$$\begin{aligned}
\mathsf{Type}_{\blacklozenge} &\triangleq ?\\
\mathbf{!}A.\, S &\triangleq ?\\
\mathbf{?}A.\, S &\triangleq ?\\
\mathtt{end} &\triangleq ?
\end{aligned}$$

Session types as a new type kind:

$$\begin{aligned} \mathsf{Type}_{\blacklozenge} &\triangleq ? \\ \mathbf{!}A.\, S &\triangleq ? \\ \mathbf{?}A.\, S &\triangleq ? \\ \texttt{end} &\triangleq ? \end{aligned} \qquad\qquad \begin{aligned} \mathsf{Type}_{\bigstar} &\triangleq \mathsf{Val} \to \mathsf{iProp} \\ \texttt{chan}\ S &\triangleq \lambda\, w.\, ? \end{aligned}$$

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Needs to capture:

- **Exclusivity** of channel endpoint ownership

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- **Delegation** of exclusive ownership types

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Needs to capture:

- **Exclusivity** of channel endpoint ownership
- **Delegation** of exclusive ownership types
- **Session fidelity** of communicated messages

Actris Dependent Separation Protocols (iProto)

Session type-inspired protocols for functional correctness

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Session type-inspired protocols for functional correctness, describing exchanges of:

- Logical variables

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	Dependent separation protocols	Session types
Example	$\mathbf{?}\,(x:\mathbb{Z})\,\langle x\rangle\{\mathrm{True}\}.\,\mathbf{!}\,(y:\mathbb{Z})\,\langle y\rangle\{y = x + 2\}.\,\mathbf{end}$	$\mathbf{?}\mathrm{Z}.\,\mathbf{!}\mathrm{Z}.\,\mathtt{end}$
Usage	$c \rightarrowtail \mathit{prot}$	$c\ :\ \mathtt{chan}\ S$

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Session type-inspired protocols for functional correctness, describing exchanges of:

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Usage	$c \rightarrowtail \mathit{prot}$	$c\ :\ \texttt{chan}\ S$

Program example:

$$\lambda\, c.\, \texttt{let}\, x = \texttt{recv}\ c\, \texttt{in}\, \texttt{send}\ c\ (x+2)$$

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Usage	$c \rightarrowtail \mathit{prot}$	$c\ :\ \mathtt{chan}\ S$

Program example:

$$(c \rightarrowtail \mathbf{?}\,(x:\mathbb{Z})\,\langle x\rangle\{\mathrm{True}\}.\,\mathbf{!}\,(y:\mathbb{Z})\,\langle y\rangle\{y = x+2\}.\,\mathbf{end}) \mathrel{-\!\!*}$$
$$\mathtt{wp}\ (\lambda\, c.\, \mathtt{let}\, x = \mathtt{recv}\ c\ \mathtt{in}\, \mathtt{send}\ c\ (x+2))\ \{\mathrm{True}\}$$

Semantic Session Types

Session types as dependent separation protocols:

$$
\begin{aligned}
\mathrm{Type}_{\blacklozenge} &\triangleq \mathrm{iProto} \\
\mathbf{!}A.\,S &\triangleq \mathbf{!}\,(v : \mathrm{Val})\,\langle v\rangle\{A\,v\}.\,S \\
\mathbf{?}A.\,S &\triangleq \mathbf{?}\,(v : \mathrm{Val})\,\langle v\rangle\{A\,v\}.\,S \\
\mathtt{end} &\triangleq \mathbf{end}
\end{aligned}
\qquad
\begin{aligned}
\mathrm{Type}_{\bigstar} &\triangleq \mathrm{Val} \to \mathrm{iProp} \\
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\end{aligned}
$$

Dependent separation protocols:

Example:	$\mathbf{?}\,(x : \mathbb{Z})\,\langle x\rangle\{\mathrm{True}\}.\,\mathbf{!}\,(y : \mathbb{Z})\,\langle y\rangle\{y = x + 2\}.\,\mathbf{end}$
Usage:	$c \rightarrowtail \mathit{prot}$

Semantic Session Types – Example Proof

Rule:

$$\Gamma, (x : \mathtt{chan}\ (?A.\,S)) \vDash \mathtt{recv}\ x : A \dashv \Gamma, (x : \mathtt{chan}\ S)$$

Proof:

```
Lemma ltyped_recv Γ x A S :
  Γ !! x = Some (chan (<??> TY A; S))%lty →
  Γ ⊨ recv x : A ⫤ ctx_cons x (chan S) Γ.
Proof.
  iIntros (HΓx%ctx_lookup_perm) "!>".
  iIntros (σ) "HΓ /=". rewrite {1}HΓx /=.
  iDestruct (ctx_ltyped_cons with "HΓ") as
    (c Hσ) "[Hc HΓ]".
  rewrite Hσ.
  wp_recv (v) as "HA".
  iFrame "HA".
  iApply ctx_ltyped_cons; eauto with iFrame.
Qed.
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Qed.
```

$$\forall \sigma.(\Gamma, (x : \texttt{chan}\ (?A.\,S)) \vDash \sigma) \mathrel{-\!\!*} \\ \quad \mathrm{wp}\ (\texttt{recv}\ x)[\sigma]\ \{w.\ (A\ w) * (\Gamma, (x : \texttt{chan}\ S) \vDash \sigma)\}$$

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Qed.
```

```
Γ : ctx Σ
x : string
A : ltty Σ
S : lsty Σ
σ : gmap string val
==============================
"HΓ" : Γ,(x:chan (<??>TY A; S))
         ⊨ σ
------------------------------∗
WP recv (σ(x))
  {{ v, A v ∗
      Γ,(x : chan S) ⊨ σ }}
```

Rule:

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Qed.
```

$$\frac{\Gamma, (x : A) \vDash \sigma}{\exists v.\ (\sigma(x) = v) * (\Gamma \vDash \sigma) * (A\ v)}$$

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σ : gmap string val
c : val
Hσ : σ(x) = c
==============================
"Hc" : c ↣ (<??> TY A; S)
"HΓ" : Γ ⊨ σ
------------------------------∗
WP recv c
  {{ v, A v ∗
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```

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$$c \rightarrowtail \mathbf{?}\,\vec{x}{:}\vec{\tau}\,\langle v\rangle\{P\}.\,\mathit{prot} \twoheadrightarrow$$
$$\texttt{wp recv}\ c\ \{w.\ \exists(\vec{y} : \vec{\tau}).\ (w = v[\vec{y}/\vec{x}]) * c \rightarrowtail \mathit{prot}[\vec{y}/\vec{x}] * P[\vec{y}/\vec{x}]\}$$

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A v ∗
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v  : val
==============================
"Hc" : c ↣ S
"HΓ" : Γ ⊨ σ
------------------------------∗
Γ,(x : chan S) ⊨ σ
```

Semantic Session Types – Example Proof

Rule:

$$\Gamma, (x : \mathtt{chan}\ (?A.\, S)) \vDash \mathtt{recv}\ x : A \dashv \Gamma, (x : \mathtt{chan}\ S)$$

Proof:

```
Lemma ltyped_recv Γ x A S :
  Γ !! x = Some (chan (<??> TY A; S))%lty →
  Γ ⊨ recv x : A ⫤ ctx_cons x (chan S) Γ.
Proof.
  iIntros (HΓx%ctx_lookup_perm) "!>".
  iIntros (σ) "HΓ /=". rewrite {1}HΓx /=.
  iDestruct (ctx_ltyped_cons with "HΓ") as
    (c H
  rewrit
  wp_rec
  iFrame "HA".
  iApply ctx_ltyped_cons; eauto with iFrame.
Qed.
```

$$\frac{\Gamma, (x : A) \vDash \sigma}{\exists v.\, (\sigma(x) = v) * (\Gamma \vDash \sigma) * (A\, v)}$$

```
Γ : ctx Σ
x : string
A : ltty Σ
S : lsty Σ
σ : gmap string val
c : val
Hσ : σ(x) = c
v : val
==============================
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  iFrame "HA".
  iApply ctx_ltyped_cons; eauto with iFrame.
Qed.
```

```
No more subgoals.
```

Extensions

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Iris and **Actris** gives immediate rise to many type features

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Session subtyping	Actris 2.0 subprotocols ($\sqsubseteq$)

Overview of Features – Definitions

Function types: $A \multimap B \triangleq \lambda\, w.\, \forall v.\, \triangleright(A\, v) \twoheadrightarrow \mathrm{wp}\ (w\ v)\ \{B\}$

Shared references: $\mathtt{ref}_{\mathtt{shr}}\ A \triangleq \lambda\, w.\, (w \in \mathrm{Loc}) * \boxed{\exists v.\, (w \mapsto v) * \square(A\, v)}$

Copyable types: $\mathtt{copy}\, A \triangleq \lambda\, w.\ \square(A\, w)$

Lock types:

$$\mathtt{mutex}\, A \triangleq \lambda\, w.\, \exists lk, \ell.\, (w = (lk, \ell)) * \mathtt{isLock}\ lk\ (\exists v.\, (\ell \mapsto u) * \triangleright(A\, v))$$
$$\overline{\mathtt{mutex}}\, A \triangleq \lambda\, w.\, \exists lk, \ell.\, (w = (lk, \ell)) * \mathtt{isLock}\ lk\ (\exists v.\, (\ell \mapsto u) * \triangleright(A\, v)) * (\ell \mapsto -)$$

Session choice:

$$\oplus\{\vec{S}\} \triangleq \mathbf{!}\,(l : \mathbb{Z})\,\langle l\rangle\big\{l \in \mathrm{dom}(\vec{S})\big\}.\, \vec{S}(l)$$
$$\&\{\vec{S}\} \triangleq \mathbf{?}\,(l : \mathbb{Z})\,\langle l\rangle\big\{l \in \mathrm{dom}(\vec{S})\big\}.\, \vec{S}(l)$$

Recursion: $\mu\,(X : k).\, K \triangleq \mu\,(X : \mathrm{Type}_k).\, K \qquad (K \text{ must be contractive in } X)$

Polymorphism:

$$\forall(X : k).\, A \triangleq \lambda\, w.\, \forall(X : \mathrm{Type}_k).\, \mathrm{wp}\ w\ ()\ \{A\}$$
$$\exists(X : k).\, A \triangleq \lambda\, w.\, \exists(X : \mathrm{Type}_k).\ \triangleright(A\, w)$$
$$\mathbf{!}_{\vec{X}:\vec{k}}\, A.\, S \triangleq \mathbf{!}\,(\vec{X} : \vec{\mathrm{Type}}_k)(v : \mathrm{Val})\,\langle v\rangle\{A\, v\}.\, S$$
$$\mathbf{?}_{\vec{X}:\vec{k}}\, A.\, S \triangleq \mathbf{?}\,(\vec{X} : \vec{\mathrm{Type}}_k)(v : \mathrm{Val})\,\langle v\rangle\{A\, v\}.\, S$$

Term subtyping: $A <: B \triangleq \forall v.\, A\ v \twoheadrightarrow B\ v$

Session subtyping: $S_1 <: S_2 \triangleq S_1 \sqsubseteq S_2$

Manual Typing Proofs

An Untypeable Program

Recall the following judgement:

$$\lambda\, c.\, (\texttt{recv}\ c \mid\mid \texttt{recv}\ c) : \texttt{chan}\ (\mathbf{?}\mathsf{Z}.\, \mathbf{?}\mathsf{Z}.\, \texttt{end}) \multimap (\mathsf{Z} \times \mathsf{Z})$$

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The rule is just another lemma proven by unfolding all type-level definitions

$$(c \rightarrowtail \mathbf{?}\,(v_1 : \mathsf{Val})\,\langle v_1\rangle\{v_1 \in \mathbb{Z}\}.\, \mathbf{?}\,(v_2 : \mathsf{Val})\,\langle v_2\rangle\{v_2 \in \mathbb{Z}\}.\, \mathsf{end}) \mathrel{-\!\!*}$$
$$\mathsf{wp}\,(\texttt{recv}\ c \parallel \texttt{recv}\ c)\,\{v.\, \exists v_1, v_2.\, (v = (v_1, v_2)) * \triangleright(v_1 \in \mathbb{Z}) * \triangleright(v_2 \in \mathbb{Z})\}$$

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And then using Iris's ghost state machinery! Beyond the scope of this talk

Concluding Remarks

Summary

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   - By exploiting the expressivity of Iris and Actris

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2. **Support for “racy” yet safe programs**
   - We extend the type system with judgments for “racy” programs like

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3. **Mechanised soundness proof of our results**
   - We mechanised it in Coq: https://gitlab.mpi-sws.org/iris/actris/-/tree/cpp21
   - By building on top of Iris and Actris frameworks and libraries
   - Artifact: https://zenodo.org/record/4322752