

MINING NATURE

$$S = \frac{\pi A k c^3}{2 h G}$$

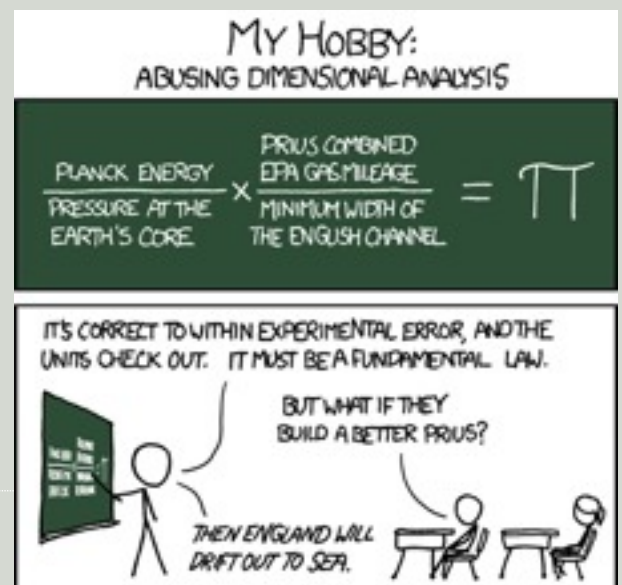
Bekenstein–Hawking entropy formula

Given a list of mathematical and physical constants, find equations involving them

Background. Equations among constants such as $E=MC^2$ can be thought of as zero sum laws: Taking logarithms we have $\log(E) - \log(M) - \log(C^2) = 0$. One can turn this around and ask: Given a set of constants, find the sets of $k=3,4,\dots$ constants that sum to zero. In turn, each such sum corresponds to an equation.

Exercise. In this implementation exercise you are given a file containing logarithms of various mathematical and physical constants, each with a textual description. Your task is to write a program that finds sets of $k \in \{3,4\}$ values that sum to zero. Because of rounding errors your implementation should initially round down values to a multiple of $2^{-20} = 1/1048576$.

- As a baseline, implement a simple program that computes the sum for each set of k values. There is a recursive solution that works for any k . (This may not run in reasonable time for $k \geq 4$.)
- Faster solution for $k=3$: Use the Threesum algorithm, e.g. as described in Algorithms 4th edition, section 1.4.
- Faster solution for $k=4$: Use a hash table to store all sums of two values (rounded to the nearest multiple of 2^{-20}). Then traverse all sums of two values, and look up the *negated* value in the hash table. This would correspond to a zero-sum of four variables. Find a condition that makes you output each such set only once.
- **Optional.** Space-efficient solution for $k=4$: Use a priority queue to generate all sums of two values in increasing order. The priority queue should contain, for each number x , the smallest sum involving x that is not yet reported (if any). Updates can be done efficiently if you store a sorted array of values, and keep track of the two values behind each sum. Similarly, generate all sums in decreasing order. Merging these lists (without storing them!) you can find all zero sums, using linear space.



<http://xkcd.com/687/>

You are given a test data file for debugging, and a larger data file for testing scalability. Each constant is marked either M (math) or P (physics). To reduce the output size, find only sums that include at least one M.

Hand-in:

- Implement algorithms for the first three tasks (optionally the 4th).
- Test your implementations on the supplied inputs and $k=3,4$. For the baseline solution and $k=4$ you may simply estimate the running time.
- Write comments in the source code of each, arguing for correctness.
- Analyze the complexity of each implementation in terms of the number n of constants.

You are allowed to do the design and implementation in pairs.