Exercises, Algorithm Design II, Fall 2012

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The case studies consider data sets that can be found via the course home page. The class will split into smaller groups that work on one of the cases, and later each group should present their algorithm design. Implementing the solution after the exercises is encouraged, and could yield components useful for the final project. It is part of the case studies to clarify what exactly should be computed, and what assumptions on data can be made.

Algorithm Design Case Study: Packing a box of primes

The file `primes1000000.txt` contain a list of all 78498 prime numbers up to 10^6. We wish to package some of these numbers in a "box" containing primes with a sum of at least 10^9, but as small as possible. The sum of all the primes is 37,550,402,023, so solutions do exist, but how close to 10^9 can we make the sum?

1. Can you find imformation about this problem, or a generalization, or related problems, on the *Compendium of NP optimization problems*?

2. Suggest a dynamic programming approach that will find an optimal solution exactly. Its time complexity will depend on the size of the numbers involved — is it polynomial time?

3. Suggest a faster approximation algorithm. You may consult chapter 11.8 in the book for inspiration if you get stuck.

Algorithm Design Case Study: Approximating TSP

We again consider the Euclidian Traveling Salesperson Problems:

`http://www.tsp.gatech.edu/world/countries.html`.

This time we will consider *tours*, i.e., paths that start and end in the same town.

1. Can you find imformation about this problem, or a generalization, or related problems, on the *Compendium of NP optimization problems*?

2. Argue that a shortest tour cannot have smaller weight than a minimum spanning tree (MST).

3. Argue that given a MST of weight W, one can easily construct a tour of weight $2W$. What kind of approximation algorithm does this lead to?

4. Design an algorithm to compute an MST for the towns in a given country. (Can you do this using linear space?) How far is the lower bound from the optimal bounds reported?

5. The tour of Sweden based on the MST has some clear possibilities for improvement, e.g., we need only visit each town one. Experiment with heuristics for improving the tour, and see how close to optimal you can get.