An Experimental Comparison of Different Heuristics for the Master Bay Plan Problem*

Daniela Ambrosino[1], Davide Anghinolfi[2],
Massimo Paolucci[2], and Anna Sciomachen[1]

[1] Department of Economics and Quantitative Methods (DIEM),
University of Genoa Via Vivaldi 5, 16126 Genova, Italy
{ambrosin,sciomach}@economia.unige.it
[2] Department of Communication, Computer and Systems Sciences (DIST),
University of Genoa Via Opera Pia 13, 16145 Genova, Italy
paolucci@dist.unige.it, davide.anghinolfi@unige.it

Abstract. Different heuristics for the problem of determining stowage plans for containerships, that is the so called Master Bay Plan Problem (MBPP), are compared. The first approach is a tabu search (TS) heuristic and it has been recently presented in literature. Two new solution procedures are proposed in this paper: a fast simple constructive loading heuristic (LH) and an ant colony optimization (ACO) algorithm.

An extensive computational experimentation performed on both random and real size instances is reported and conclusions on the appropriateness of the tested approaches for the MBPP are drawn.

Keywords: heuristics, metaheuristics, stowage plans.

1 Introduction and Problem Definition

The stowage of containers on a ship, that is the so called master bay plan problem (MBPP), is faced daily by each terminal management. This problem can be defined as follows: given a set C of n containers of different types to load on a ship and a set S of m available locations within the ship, we have to determine the assignment of the containers to the ship locations in order to satisfy the given structural and operational constraints related to both the ship and the containers and to minimise the total loading time.

Set C is partitioned into two subsets, namely T and F, consisting of 20 and 40 feet (20' and 40') containers, respectively. Each location is addressed by three indices, i, j and k, representing its bay (i), row (j) and tier (k) position in the ship; let I, J and K be, respectively, the corresponding set of bays, rows and tiers available for the stowage. Moreover, let $E \subset I$ be the subset of even bays, that are used for stowing 40'containers,

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and $O \subset I$ the one of odd bays, that are used for stowing 20'containers. Tiers in the hold and on the deck are denoted by K^H and K^D, respectively. Finally, the locations having the same pair of bay and row identify a *stack*; let hence P be the set of all the available stacks in the ship.

In this paper the MBPP involves only the loading decisions at the first port without taking into account possible loading operations at the next ports of the ship route. We assume that the container handling operations are performed by quay cranes, which are positioned on the quay side of the ship. In [12], the problem of defining stowage plans is split into a two-step process concerning first the shipping line and then the terminal management; the authors provide for each step a review of the corresponding optimization models. A relevant literature update is provided in [11].

In this work we also assume that the ship is empty and only one yard crane is available for handling the containers at the quay. In spite of such simplifying assumptions, the resulting mathematical formulation is not simple. In fact, in order to model the MBPP we must deal with the assignment constraints (i.e., each container must be assigned to at most one location and each location must receive at most one container) and the knapsack constraint (i.e. the total weight of the containers loaded on the ship cannot exceed the ship capacity); furthermore, besides these basic combinatorial constraints, some constraints related to the size, weight and destination of the containers, together with others related to the ship stability, have to be taken into account. Stability conditions involve three different types of equilibrium: *horizontal, cross* and *vertical.*

MBPP is NP-Hard [5]. Some Integer Programming models for MBPP are proposed in [6] and [10]. Unfortunately these papers deal with simplified version of problem so that the proposed models are not suitable for real life large scale applications. MBPP is described in details in [1] where a 0/1 Integer Programming (IP) model is presented. In that model the decision variables are related to the assignment of containers to locations of the ship; the model is used for solving up to optimality only very small instances. Successively, in [3] a new 0/1 IP model is proposed where variables correspond to the assignment of ship locations to groups of containers, each one characterized by range of weight (e.g., low, medium or high), type and destination; using that model the authors solve larger instances, even if not always integer feasible solutions are found within a reasonable CPU time, that is some hours. In [3] MBPP is solved by using a three step approach. First a *bay assignment* procedure is performed to assign subsets of containers with the same destination to predefined subsets of bays; successively for each partition of the ship a single destination 0/1 IP model is considered, where the ship stability constraints are relaxed. Finally, possible infeasibilities of the global solution due to the violation of either cross or horizontal stability conditions are removed by means of a tabu search procedure. The tabu search algorithm also tries to improve the global solution, i.e., to reduce the total loading time. The main features of the tabu search presented in [3] is that it is based on seven classes of moves which combine three kinds of items that can be moved, that is a single container, a stack of containers and a bay, with three kinds of position exchanges, that is anterior-posterior location exchange, left side-right side exchange, and cross exchange.

By using such heuristic method it is possible to check and force feasibility up to small- medium sized instances, while for larger instances, it is not possible to use the 0/1 IP model for obtaining an initial solution. For this reason we propose a simple constructive procedure that is described in Section 2. Moreover, in this paper we present an Ant Colony Optimization (ACO) heuristic, described in Section 3, that is successfully applied to very large instances. Section 4 reports the performed experimental analysis; finally, conclusions are drawn in Section 5.

2 Simple Constructive Heuristic

In this section we describe a new solution method, that is a simple constructive loading heuristic (LH). LH determines a solution for MBPP that satisfies both the destination and the weight constraints in the following steps.

1. Let $C' = \{ C_1 , C_2 ,\ldots C_h,\ldots. C_D\}$ be a partition of C, where C_h denotes the set of containers having as destination port h. Split C_h according to the type of containers, that is 20' and 40', thus obtaining sets C_{hT} and C_{hF}, respectively.
2. Apply the bay partitioning procedure described before (see [3] for more details) to determine the subsets $I_h \subseteq I$ of bays assigned to destination h, $h = 1, \ldots,D$.
3. For each destination h, starting from the last one (D) back to 1, assign first containers belonging to C_{hT} and then containers belonging to C_{hF} as follows.
   - 3.1 Sort C_{hX} (where $X \in \{T,F\}$) in increasing order of weights.
   - 3.2 Repeat - Until $C_{hX} \neq \emptyset$ or $I_{hX} = \emptyset$
     1. Select $i \in I_{hX}$
     2. For k=1 to K and j=1 to J assign container $c \in C_{hX}$ to location (i, j, k), starting from the first container in the set (i.e., the heaviest one), then set $C_{hX} = C_{hX} \setminus \{c\}$ and $I_{hX} = I_{hX} \setminus \{i\}$
   - 3.3 If $C_{hX} \neq \emptyset$ and $I_{hX} = \emptyset$ try to locate the remaining containers without violating the destination and weight constraints as follows
     1. If $C_{hT} \neq \emptyset$, the remaining containers are possibly located above 20' containers having destination $h' > h$ without violating the weight constraints.
     2. If $C_{hF} \neq \emptyset$, the remaining containers are possibly located above 20' containers if they have the same destination; otherwise, either above 40' or 20' containers having destination $h' > h$, provided that the weight constraints are satisfied.

3 The Proposed ACO Approach for the MBPP

In this section we describe the main aspects of an ant colony optimization (ACO) approach for MBPP. ACO is a population-based metaheuristic which tries to emulate the successful behaviour of real ants cooperating to find shortest paths to food for solving combinatorial problems ([7], [9]). Real ants have an effective indirect way to communicate each other which is the most promising trail towards food: ants produce a natural essence, called pheromone, which they leave on the followed trail to food in order to mark it. The pheromone trail evaporates with time and it disappears on the paths left by the ants; however, it can be reinforced by the passage of further ants: thus, effective (i.e., shortest) paths leading to food are finally characterized by a strong pheromone track, such that shortest paths are followed by most ants. The ACO metaheuristic has been both the subject of theoretical studies and successfully applied to many combinatorial optimization problems.

We consider a set of m artificial ants. At each iteration the ants progressively fix the destination, type and class of weight for the available ship locations; thus they construct a solution by assigning the containers to the locations, accordingly. After that, a Local Search (LS) is executed starting from the best solution found in the current iteration, a global pheromone update phase takes place and the whole process is iterated. The algorithm terminates when a maximum number of iterations is reached.

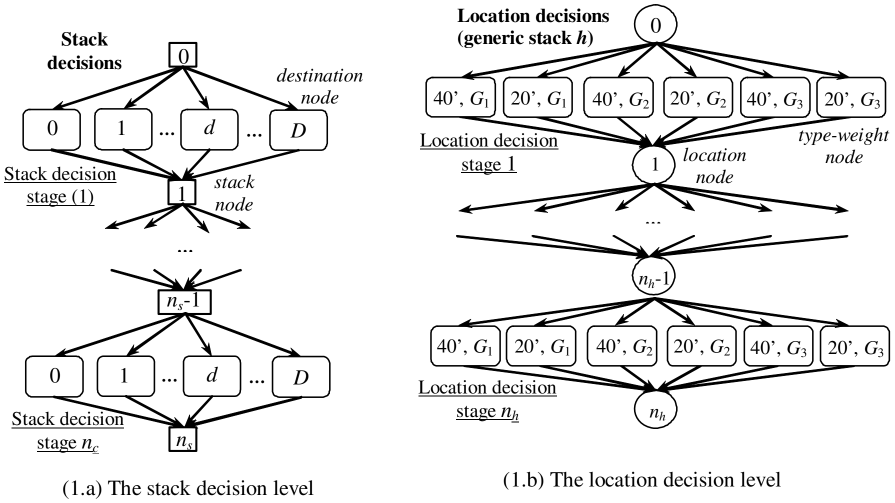
The proposed ACO is the adaptation of the algorithm introduced in [4]. Such an approach is inspired by the Ant Colony System (ACS) [8] and *Max-Min* Ant System (MMAS) [13] and it includes a new local and global pheromone update mechanism. For the sake of brevity, hereinafter we focus only on the modelling aspects highlighting how the ACO approach can be applied to MBPP. Details of the algorithm can be found in [4], while readers interested in a comprehensive presentation of the ACO metaheuristic can find a valuable and general reference in [7].

Let $CD=\{1,\ldots,D\}$ denote the set of destinations for the containers in C ordered according to the ship route. At each iteration the ants take a sequence of decisions nested in two levels: *stack decision level* and *location decision level.* At the higher stack decision level the ants consider a stack at a time and fix the latest destination for the containers loaded in its locations (e.g., if d^{max} is the latest destination for a stack, then only containers bound for $1,\ldots,d^{max}$ can be loaded in it). At the lower location decision level, the ants establish the type $t\in\{\mathrm{T},\mathrm{F}\}$ and class of weight $g\in G$ for each location in the considered stack, finally assigning to it a container with a compatible destination, type and class of weight, which satisfy both destination and vertical equilibrium constraints.

Such decisions correspond to the search of a path from a start node (the ant colony *nest*) to an end node (the *food*) in a *construction graph* structured in two nested levels, as depicted in Figure 1.

Figure 1.a shows the graph corresponding to the stack decision level; here the ants determine a path from the stack node 0 (the ant nest) to the stack node n_s (associated with the last considered stack), selecting at each stage one destination in $\mathrm{CD}\cup\{0\}$, where 0 denotes that the stack is not assigned. The order according to which stacks are considered in the stack decision stages is heuristically fixed for both favouring the ship stability and reducing the loading time. In particular, we built a sorted list of rows in increasing order of the associated loading time t_j, $j\in J$ (computed as $t_j=\max_{k\in K} t_{jk}$, being t_{jk}, $j\in J$, $k\in K$, the loading time for row j and tier k) but alternating a left and a right row. Similarly, we build a sorted list of bays alternating a bay in the middle of the ship, one in the anterior and one in the posterior part with a procedure similar to the pre-assignment performed in [2]. Finally, the sequence of stacks defining the stack decision stages is produced by extracting the index i and the index j respectively from the sorted lists of bays and rows.

After a stack level decision the ants proceed considering the locations in the stack. Figure 1.b details the location decision level for a generic stack h. Also in this case the ants' decisions correspond to a path from the location node 0 to the location node n_h (associated with the last available location in stack h), determining at each stage the type and class of weight (here $G=\{G_1, G_2, G_3\}$, where G_1 stands for light, G_2 for medium and G_3 for heavy) for the location so that no vertical equilibrium constraint



(1.a) The stack decision level

(1.b) The location decision level

Fig. 1. The construction graph of the ACO algorithm

or type compatibility is violated. In the following, we denote by N_s, N_d, N_l and N_{tw} respectively the sets of stack, destination, locations and type-weight nodes. Note that (a) in the stack decision level of the construction graph the stage associated with a stack on deck immediately follows the corresponding stack in hold; (b) in the location decision level the location nodes are ordered from the lowest to the higher. Once both the latest destination and the type and class of weight for a location are fixed at a location decision stage, the ants actually assign to it a specific compatible container selected from the not yet loaded ones that do not violate any destination constraint (this is simply obtained sorting the available compatible containers in decreasing order of destination).

Note that n_s may be greater than the number of available stacks on the ship since the stacks with available locations may be reconsidered whenever there are not assigned containers. We must remark that the number of stages considered at the location decision level may vary since the ants' decisions at a stage force the available alternatives at the successive stages: in particular, a single type of container is allowed for a stack, and weight and destination constraints can prohibit certain assignment patterns. Therefore, assuming n_s=|P|, we have $2 \cdot |I| \cdot |J|$ stack decision stages and at most $|K^H|$ and $|K^D|$ location decision stages for the locations associated with a stack in the hold and on the deck respectively, which in turn consist of $3 \cdot |K^H|$ and $3 \cdot |K^D|$ pairs of type-weight nodes.

The information about the ants state are (a) the sets of containers that remain to be located $C_R=\{(p, t, g, nc_{ptg}): p \in CD, t \in \{20', 40'\}, g \in G\}$, where nc_{pgt} is the number of containers to be loaded with destination p, type t and group of weight g; (b) the partial set of decisions for stow locations $C_A=\{(i, j, k, p, t, g): i \in I, j \in J, k \in K, p \in D, t \in \{20', 40'\}, g \in G\}$. A pheromone trail is associated with a subset U of the arcs of the construction graph: U includes the arcs (s, d): $s \in N_s$, $d \in N_d$ connecting stack nodes to destination nodes at the stack decision level, and the arcs (*l, z*): $l \in N_l$, $z \in N_{tw}$ connecting location nodes to type-weigh nodes at the location decision level.

The ant solution construction process outlined so far is quite flexible as it allows to locate containers with different destinations in the same stack or bay. The selection of a destination node and type-weigh node at each stage of the two decision levels is performed in two steps similarly to the ACS: first an ant determines the node selection rule between *exploitation* and *exploration*, then it actually selects the node. The ant extracts a random uniform number $q\sim[0,1]$ and chooses exploitation if $q \le q_0$ (where $q_0 \in [0,1]$ is a fixed parameter), otherwise exploration. The *exploitation* rule at a generic decision node *s* such that $(s, h) \in U$ selects the next node h^* in a deterministic way as

$$h^* = \arg \max_{h:(s,h)\in U} \{\tau_e(s,h) \cdot [\eta(s,h)]^\beta\} \tag{1}$$

whereas the *exploration* rule according to a *selection probability* $\pi_e(s,h)$ computed as

$$\pi_e(s,h) = \frac{\tau_e(s,h) \cdot [\eta(s,h)]^\beta}{\sum\limits_{z:(s,z)\in U} \tau_e(s,z) \cdot [\eta(s,z)]^\beta} \tag{2}$$

The subset of arcs $(s, h) \in U$ identifies the solution components and $\tau_e(s,h)$ is the pheromone trail associated with the component (*s*, *h*) at iteration *e*. The pheromone trails represent the ACO learning device and provide a measure of the appropriateness of selecting a component during the construction of "good" solutions. Following the same approach in [4], the pheromone values assigned to $\tau_e(s,h)$ are independent of the objective function values associated with previously explored solutions including the component (*s*, *h*); pheromone values vary in an arbitrary range $[\tau_{Min}, \tau_{Max}]$, with $\tau_{Min} < \tau_{Max}$, which is fixed independently of the specific problem or instance considered (actually the algorithm behaviour does not depend on the choice of τ_{Max} and τ_{Min}). The pheromone trails are initialized as $\tau_0(s,h) = (\tau_{Max} + \tau_{Min})/2$ and their variation in the range $[\tau_{Min}, \tau_{Max}]$ during the exploration process, i.e., the ant colony learning mechanism, is controlled by the same global pheromone update rule proposed in [4] that allows a smooth variation of $\tau_e(s,h)$ within these bounds such that both extremes are asymptotically reached. The quantity $\eta(s,h)$, associated with the component (*s*, *h*), is a heuristic value that is used to direct the ants' selections during the first iterations when the pheromone trails are almost the same for all the components. We based the computation of $\eta(s,h)$ on a very simple rule which returns $\eta(s,h) = \eta_t(s,h) \cdot \eta_c(s,h)$, where $\eta_t(s,h)$ and $\eta_c(s,h)$ are the heuristic values relevant to the choice of the type of containers (i.e., the use of an even bay or the two paired odd bays) and of the class of weight for the location associated with *h* respectively. In order to minimize the loading time we favour the assignment of 20' containers to the locations in the rows closer to the berth side, fixing $\eta_{20}(s,h) = 2$ for the relevant nodes in the construction graph, as well as the assignment of the heaviest containers to the lowest tiers in the hold and on the deck, fixing $\eta_t(s,h) = 2$ for the relevant nodes, letting $\eta_t(s,h) = \eta_g(s,h) = 1$ in all the other cases. The quantity β in (1) and (2) is a parameter representing the importance of the heuristic value with respect to the pheromone trail in the ant selection rules.

Whenever an ant reaches the final state node n_s a tentative solution x for MBPP is build. Tentative solutions could not be feasible as some equilibrium constraints could be violated and some containers could be not loaded. Then, we compute the global cost for the solution as

$$Z(x) = M_s(\sigma_1(x) + \sigma_2(x)) + M_{nl}\mu(x) + L(x) \tag{3}$$

being $\mu(x)$ the number of not loaded containers and M_s and M_{nl} two penalties for the violation of stability constraints and for not loaded containers in x, respectively. After all ants a=1,..., m have determined a solution x_e^a at an iteration e, the best solution found in the iteration, i.e., $x_e^{best} = \arg\min_a Z(x_e^a)$, is determined and a LS step takes place. The purpose of this LS is that of perturbing x_e^{best} by means of a set of moves that change the locations of a subset of loaded containers in order to eliminate the possible violation of stability constraints and to improve the overall loading time. The LS procedure at each iteration performs the following sequence of three types of random moves, whose details are provided in [3]: Anterior-Posterior exchange of containers (APC); Left-Right side exchange of containers (LRC); Cross exchange of containers (CEC). As regards the solution feasibility, we should note that the LS is devoted only to recover violations of stability which, emerging from the solutions considered as a whole, are difficult to avoid during the ACO solution construction process. Thus, the objective function minimized by the LS disregards the $M_{nl}\mu(x)$ component since, leaving in this way the task of loading all the required containers to the ACO solution construction process. The LS adopts the first improvement move acceptance rule and it terminates after a fixed maximum number of iterations. However, whenever the overall ACO algorithm reaches a maximum number of non improving iterations, the LS maximum number of iterations is temporarily doubled until an improved solution is found.

Finally, the pheromone trails associated with the solution components included in the best solution found so far are reinforced while the other components are evaporated according to the global pheromone update rule introduced in [4], and the algorithm iterates.

4 Experimental Results

The proposed MBPP algorithms were coded in C++, using the commercial Cplex 9.0 as 0/1 IP solver, and tested on a 1.5GHz, Intel Celeron PC with 1Gb RAM. The tests were related to two containerships of different sizes. The first containership is the medium size European Senator, whose data have been provided by the SECH Terminal of Genova, Italy. The European Senator has a 2124 TEU capacity and is composed by 17 odd bays, 10 rows and 6 tiers in the hold and 21 odd bays, 12 rows and 5

tiers in the upper deck. Then we consider a larger containership with a capacity of 5632 TEUs.
The considered instances are characterized by different level of occupancy of the ship and different type of containers to load:

- the total number of containers (*TEU* and absolute number) ranges from 945 to 1800 (TEUs) and 715 to 1413 containers for instances with 2 or 3 destinations, and from 4920 to 5510 (TEUs) and 3800 to 4110 containers for instances with 4 or 5 destinations. The level of occupancy ranges from 50% to 97%;
- the percentage of 20' and 40' containers are 70% - 30% and 60% - 40%, respectively, for instances with 2/4 and 3/5 destinations;
- the percentage of containers for three groups of weight ranges from 30% - 60% - 10% to 40% - 35% - 15%;
- the partition of containers for each destination is 50% - 50% and 30 % 35% 25%, respectively, for instances with 2 or 3 destinations, whilst containers are uniformally distributed in case of 4 and 5 destinations.

The loading times depend on the row and tier and grow from left side rows to the right ones and from highest tiers on the deck to the lowest ones in the hold; times are expressed in 1/100 of minute and range from 120 to 330. We first tried to find a global optimal solution using the exact 0/1 IP model given in [3]. We fixed as termination conditions for the solver an absolute gap = $5 \cdot Nd$ minutes, where Nd is the number of destinations of the considered instance, and a maximum time limit of 1 hour, and we obtained the following results. Medium size ship:

- no integer solution was found in one hour of computation for all the 4 instances with 3 destinations;
- the solver stopped with a feasible solution after reaching the maximum time limit for 5 instances out of 10 with 2 destinations;
- the solver required on the average after 16m and 47s to terminate for the remaining 5 instances with 2 destinations.

Large size ship: no integer solution was found even extending the maximum computation time to two hours.

Note that finding an exact solution for the instances with more than two destinations was significantly hard; therefore, the need of an effective heuristic for medium and large containerships is apparent.

We tested the TS and the ACO approaches comparing the obtained results also with the ones produced by the exact 0/1 IP model for medium size instances and by the simple constructive LH followed by the TS (LH-TS) for large size instances.

We performed some preliminary tests to select suitable values for the TS and the ACO parameters, identifying the following configurations: for the TS we fixed tenure = 80, diversification after 30 non improving iterations, diversification length = 35 iterations, termination conditions corresponding to maximum number of iterations = 500 and maximum not improving iterations = 50. For the ACO we used 80 ants, the pheromone evaporation factor α=0.05, q_0=0.95, β=1, maximum number of non improving iterations = 8 and maximum number of iterations =1000. Since random choices are used in both TS and ACO, five independent runs were performed for each instance and the average results were considered.

Table 1 shows the results for the medium size test instances. We report in the first group of columns (*Exact 0/1 IP*) the results produced by the exact 0/1 IP model, followed by three groups of columns showing respectively the starting solutions of the TS obtained by solving the 0/1 IP model for the single destination MBPP (*SD-MBPP*), the final solutions yielded by the TS (*Tabu Search*) and the final solutions produced by the ACO. An absolute gap of 5m was fixed as the termination condition for the SD-MBPP 0/1 IP model and the loading times shown in the columns *Obj* are expressed as 1/100 of a minute. Note that *Time (a)* and *Time (b)* columns report the CPU times in seconds needed by the 0/1 IP solver respectively for the exact model and the single destination model, whereas the *Avg Time* columns for the TS and ACO show the total CPU times needed by the two-heuristic approaches. The last table row finally shows the overall average CPU times for this set of instances.

First, we must remark that both the TS and the ACO approaches were able to find solutions satisfying both the cross and the horizontal stability conditions on every run for each instance. The average CPU times in Table 1 show that both heuristics were able to find feasible solutions in a fraction of the time needed by the exact 0/1 IP

Table 1. The results for the medium size ship

		Exact 0/1 IP (max time=1h, absolute gap=5×N_d m)		**SD-MBPP (absolute gap=5m)**				**Tabu Search (average over 5 runs)**		**ACO (average over 5 runs)**	
Ist.	**Nd**	**Obj (a)**	**Time (a)**	**Obj (b)**	**σ_1**	**σ_2**	**Time (b)**	**Avg obj**	**Avg Time**	**Avg obj**	**Avg Time**
1	2	139530	1300.8	150570	15	5380	68.4	142990	101.1	144162	142.0
2	2	149440	1606.4	155100	20	1065	11.6	151700	43.8	154918	158.6
3	2	161670	1876.7	168310	0	1225	7.4	164444	40.5	167780	180.1
4	2	178320	1613.3	186310	0	1435	21.8	180906	54.3	183336	174.0
5	2	219650	3698.6	225510	0	1115	17.6	222600	58.1	227486	189.2
6	2	197410	1999.8	203040	0	1555	18.0	200072	49.7	203566	191.6
7	2	213520	3698.5	219330	0	1235	12.1	216036	52.0	221572	154.9
8	2	244660	3698.4	252640	0	1495	21.3	247810	63.5	254168	141.8
9	2	248050	3295.0	253640	5	1010	212.3	250128	243.4	258578	137.9
10	2	293150	3704.7	293310	0	1960	17.0	293530	41.5	305228	115.7
11	3	-	3600.0	206590	20	5685	65.0	200140	119.6	204452	168.2
12	3	-	3600.0	220210	15	1300	7.5	217660	45.5	224822	181.2
13	3	-	3600.0	237050	0	1625	19.8	233608	65.9	239912	183.7
14	3	-	3600.0	278780	0	470	30.8	275964	66.7	285948	176.3
		Averages	2953.4				37.9		74.7		167.7

model to find the optimal; note that the CPU time consumed by the exact 0/1 model for obtaining for each instance the first suboptimal solution not worse than the average one produced by the TS is more than 10 times the CPU of the TS (in particular, an average of 846s compared to 74.8s). We compare the results of instances 1-10 in Table 1 by computing the percentage deviations of the solutions yield by the SD-MBPP model and the TS and ACO approaches from the ones found by the exact 0/1 IP model. The solutions obtained by the SD-MBPP model are worse but not very far from the ones yielded by the exact 0/1 IP model: the average percentage deviation of 3.41%, which corresponds to an average difference in the total loading time = 1h 2m 21s, was obtained with a very short average CPU time (37.9s); however, we must remark that the SD-MBPP was never able to satisfy the stability conditions (i.e., $\sigma_1,\sigma_2 > 0$), especially the horizontal one. The best results are apparently due to the TS approach that was only 1.33% worse (corresponding to an average difference in the total loading time = 24m 49s) than the exact 0/1 IP model that needed a very much longer computation. Compared to TS, the designed ACO algorithm presents worse performances with respect to average loading times and computation requirements.

For the instances with 3 destinations (11-14 of Table 1) we compared results obtained by TS and ACO, observing that the loading times produced by the ACO were worse on average of 2.94% than the ones obtained by the TS.

The results obtained for the large size instances are reported in Tables 2 and 3. As for large instances with more than 3 destinations, the ACO algorithm turned out to be the best one, and in some cases it was the only approach able to generate a solution. Note that for these instances the exact 0/1LP model never was able to determine a feasible integer solution. Since also the SD-MBPP model failed to find a feasible solution for some of the instances with a larger number of containers, we were not able to initialize the TS (in particular, is true for the instances 4 and 6 denoted with (*) in Table 2). For this reason we decided to implement the LH described in Section 2. Tables 2 and 3 show the results produced by the two steps of the TS, LH-TS and those produced by the ACO algorithm. In both tables, the first group of columns reports the characteristics of the initial solutions obtained, respectively, by the SD-MBPP model (*SD-MBPP*) in Table 2 and by the LH (*LH*) in Table 3; then, the second group of columns shows the final average results, respectively, for the TS approach (*TS*) in Table 2 and the LH-TS one (*LH-TS*) in Table 3. Note that, as the tabu search is only able to improve the stability and the loading time of a solution, the number of non loaded containers (*NLC*) in the initial solutions found by the SD-MBPP model and LH is not modified in the final TS and LH-TS solutions. The last column of Table 5 shows the non loaded containers (NLC) in the final ACO solution together with the loading time and the CPU time. Tables 2 and 3 report the stability violations (σ_1 and σ_2) only for the initial solutions as no stability violation was found in the final solutions; this is true also for the ACO solutions.The LT (i) and LT (f) columns show the loading times, respectively, for the initial and final solutions, and the LT %var column reports the percentage variation of the loading time in the final solutions with respect to the initial ones. Finally, Table 2 and 3 include three CPU times expressed in seconds: the one needed by the SD-MBPP model (CPU), the time required by the tabu search (CPU TS) and the overall (initialization plus search) cumulative time (CPU TOT). Note that the averages in the last row of Table 2 are computed without instances 4 and 6. The ACO needed less CPU time than the TS procedure, always being able to find a feasible solution. However, the ACO approach was also able to completely locate all the containers only in one case out of 11. The number of non loaded containers is lower than those obtained by TS.

Table 2. Tabu Search solutions for the large size tests

SD-MBPP							TS (average over 5 runs)			
Ist.	**Nd**	**NLC**	**LT (i)**	**σ_1**	**σ_2**	**CPU**	**LT (f)**	**LT %var**	**CPU TS**	**CPU TOT**
1	4	5	868400	0	40	14400.0	868388	0.00	25.9	14425.9
2	4	0	852590	0	735	14400.0	853002	0.05	100.4	14500.4
3	4	100	850070	0	335	14400.0	850194	0.01	33.0	14500.4
4(*)	4	84	896760	-	-	-	-	-	-	-
5	4	92	906310	5	300	11099.3	906288	0.00	22.3	11121.7
6(*)	4	85	915329	-	-	-	-	-	-	-
7	5	112	918920	20	425	7325.2	918996	0.01	48.5	7373.7
8	5	47	901750	0	270	4181.7	901760	0.00	30.5	4212.2
9	5	194	733240	35	150	10187.8	732640	-0.08	37.4	10225.1
10	5	213	727830	20	85	14953.3	727510	-0.04	40.2	14993.5
11	5	91	842650	5	60	157.3	842616	0.00	25.3	182.6
Avg.		95	844640			10122.7	844599		40.38	10163.2

Table 3. LH-TS and ACO solutions for the large size tests

	LH					LH-TS				ACO		
ist	**NLC**	**LT (i)**	**σ_1**	**σ_2**	**CPU**	**LT (f)**	**LT %var**	**CPU TS**	**CPU TOT**	**NLC**	**LT**	**CPU**
1	240	897400	140	175	10	833640	-7.10	91.99	102.0	102.0	879162	154.8
2	188	896860	50	425	14	841420	-6.18	144.8	158.8	77.4	896676	123.4
3	424	855200	140	435	10	792606	-7.32	94.7	104.7	321.0	851946	100.9
4	106	972000	90	695	12	926130	-4.72	92.0	104.0	55.6	964970	73.1
5	44	991190	395	4080	15	969842	-2.15	80.5	95.5	0.0	986150	195.9
6	95	993200	25	4100	12	965824	-2.76	85.5	97.5	65.2	978940	130.1
7	100	996540	360	2975	14	946722	-5.00	160.8	174.8	19.6	999440	201.4
8	90	966160	230	515	10	935508	-3.17	94.7	104.7	1.4	979046	109.1
9	195	820640	80	1425	12	765732	-6.69	107.3	119.3	162.6	787590	166.7
10	219	815170	130	2315	11	758280	-6.98	107.5	118.5	152.6	799750	169.9
11	123	907340	625	660	10	882852	-2.70	95.6	105.6	40.0	918210	78.7
Avg	165.8	919245			11.8	874414.8		105.0	116.8	90.67	912898	139.4

Finally, comparing the results obtained by applying the proposed approaches to large size instances, in term of percentage number of non loaded containers (NLC) and CPU time, we noted that none of the proposed approaches was able to load on board all containers: as the large ship test instances were randomly generated with occupancy level between about 87% and 97% to stress the solution capacity of the compared algorithms, we cannot claim that a solution where all the containers are loaded exists for this benchmark. What we can observe for the large size tests is that all the proposed methods produced feasible solutions in terms of stability conditions, and the ACO approach behaved better since it required lower CPU time and produced a lower average NLC.

5 Conclusions

In this paper different solution methods for the NP-hard MBPP are presented and compared. The results obtained from an extensive computational experimentation, based on both real size instances and random ones, enable us to derive two main conclusions. First, for very large size instances the ACO approach is recommended among the proposed ones. Second, the performance of the TS based approaches is strongly depending on the quality of the initial solution. In particular, for medium size instances, where it is possible to apply the TS to a good initial solution, the resulting final solutions got by the TS are very good; this is not the case for large size instances.

Starting from the results discussed in this paper, the authors intend to extend their analysis to the multi-port MBPP. Finally, in this future development the assumption about the availability of only one quay crane will be removed.

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