Formal Definition of the Container Vessel Stowage Problem
CONFIDENTIAL WORK DRAFT

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May 1, 2006

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1 Introduction

The purpose of this document is to give an accurate definition of the container vessel stowage problem. The first four sections give background information on the operation of container terminals and container vessels. Section 2 describes the organization and operation of container terminals. Section 3 gives an overview of different container types. Section 4 describes the physical arrangement and operation of deep-sea cellular container vessels.

We then turn to defining the container vessel stowage problem and start in Section 5 by introducing a vessel state. Section 6 and Section 7 define the costs and constraints of the problem. Finally, Section 8 defines valid and optimal solutions to problem.

2 Container Terminals

Today over 60% of the world's deep-sea general cargo is transported in containers. The amount of containerized cargo almost doubled from 1985 to 1995 and is expected to grow fast in the future [6].

A deep-sea container vessel typically makes round-trips between ports of two trade zones. Each voyage cycles through the ports of a *rotation string*. A particular port may be visited several times on a rotation. For instance, on the rotation between Europe and Asia shown in Figure 1, Hong Kong is visited twice.

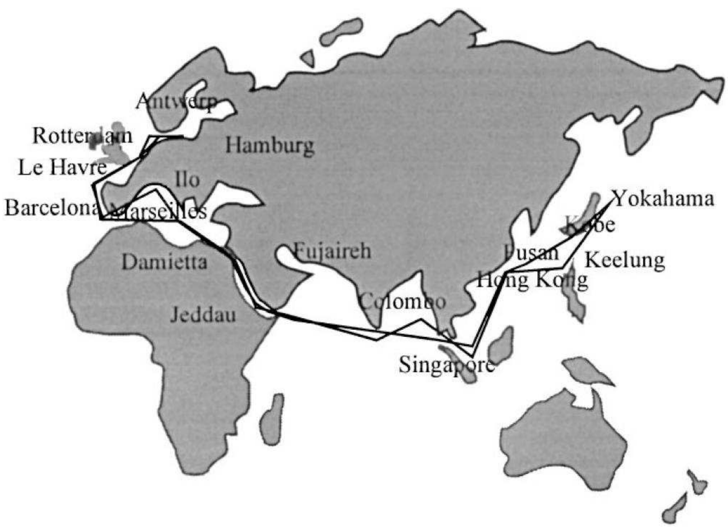
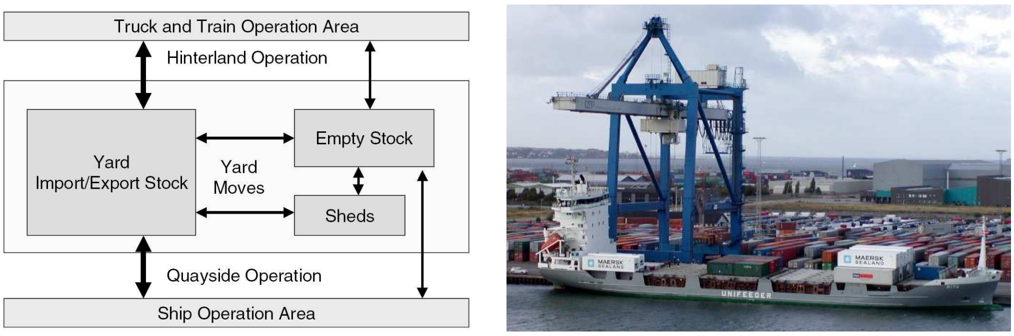


Figure 1: Rotation string example [11].

Container vessels are loaded and unloaded at container terminals. Container terminals can be described as open systems of material flow with two external interfaces. These interfaces are the *quayside* with loading and unloading of ships, and the *landside* where containers are loaded and unloaded on/off trucks and trains. Containers are stored in stacks thus facilitating the decoupling of quayside and landside operations [6].

After arrival at the port, a container vessel is assigned to a berth equipped with cranes to load and unload containers. Unloaded import containers are transported to yard positions near to the place where they will be transshipped next. Containers arriving by road or railway at the terminal are handled within the truck and train operation areas. They are picked up by the internal equipment and distributed to the respective stocks in the yard. Additional moves are performed if sheds and/or empty stocks exists within the terminal; these moves encompass the transports between empty stock, packing center, and import and export container stocks (Figure 2(a)). Quayside



(a) (b)

Figure 2: (a) Operation areas of a seaport container terminal and flow transports [6]. (b) Feeder vessel loaded in Copenhagen harbor [8].

operation or container transshipment as well as the container movement to and from the wharf are sometimes referred to as *waterside transhipment processes*. Correspondingly, one may find the terms *hinterland transhipment process* and *landside transhipment processes* [6].

Different types of ships have to be served at the quayside. The most important ones are deep-sea vessels with a loading capacity of up to 8.000 *twenty feet equivalent units* (TEU)[1]. Loading of about 2.000 boxes is common in large ports; the same is valid for unloading. Feeder vessels with capacity of 100 to 1.200 TEU link smaller regional ports with the oversea ports delivering containers for deep-sea vessels. Inland barges are used to transport containers into the hinterland on rivers and channels. Functionally, barges are means of hinterland transportation (like trucks and trains), operationally they are ships which are served by quay cranes [6]. A feeder vessel loaded by a single quay crane in Copenhagen harbor is shown in Figure 2(b).

Trucks have a capacity of up to three TEU. At container terminals they are directed to transfer points where they are loaded and unloaded. To serve trains, railway stations with several tracks may be part of container terminals. The capacity of one train is about 120 TEU [6].

The container storage area is usually separated into different stacks (or blocks) which are differentiated into rows, bays and tiers. Some stack areas are reserved for special containers like reefers which need electrical connection, dangerous goods, or overheight/overwidth containers which do not allow for normal stacking. Often stacks are separated into areas for export, import, special, and empty containers [6].

Quay cranes have a technical performance in the range of 50-60 boxes per hour, while in operation the performance is in the range of 22-30 boxes. The transport between quay and stack can

[1]Similarly we have a *fourty foot equivalent unit* (FEU). 2 TEU = 1 FEU.

be performed either by trucks with trailers, multi-trailers, Autonomous Guided Vehicles (AGVs) or straddle carriers [6]. A schematic side view of a container terminal system is shown in Figure 3 [6].

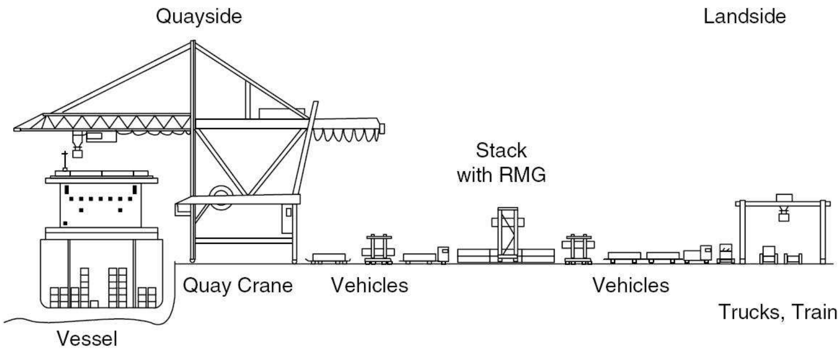


Figure 3: Container terminal system [6].

A container *move* is a re-location of a container from the vessel to the stack or vice versa. The cost of a move varies substantially between ports in the range 30-200$ [5].

Most of the processes occurring at container terminals cannot be foreseen for a longer time span. For instance, although data of containers to be delivered to terminals by trucks may be pre-advised by EDIFACT (electronic data interchange for administration, commerce and transport), the exact time when the containers arrive at the terminal is not known[2] [6].

For ship loading and unloading, although in general data of containers and their positions within the ship are precisely known in advance and the preplanning process allows the calculation of job sequences, they often have to be changed because of operational disturbances. As vessels are not static and move permanently (because of tide, weather, stability), containers which are next in the sequence cannot be accessed by the crane's spreader. Crane drivers make their own decisions and may alter the pre-calculated loading and unloading sequence by themselves [6].

3 Containers

The ISO 668 standard defines 20 and 40 feet long containers. The dimensions of 20' and 40' containers are given in Figure 4. The weight of an empty container ranges from 2 to 3.5 tons [1]. The major container types are shown in Figure 5.

- **Standard containers**. Standard containers are in the ISO 668 dimensions and are for general purpose dry cargo. Standard containers varies in their side-door configurations.
- **High-cube containers**. High-cube containers has height 9'6" instead of 8'6" of the ISO standard. They are suitable for light weight cargo.
- **Hard-top containers**. Hard-top containers have a removable roof. They typically used for heavy cargo and tall cargo, or cargo that is easy to load from the top.

[2]This is true for the north-west European ports, while East-Asian ports commonly prescribe a time-window of only several minutes when a truck has to enter a terminal.

	Dimensions	
	$20' \times 8' \times 8'6''$	$40' \times 8' \times 8'6''$
Length (meters)	5.9	12.0
Width (meters)	2.4	2.4
Height (meters)	2.6	2.6
Cubic capacity (cubic meters)	32.9	67.0
Stacking capacity	9 high	9 high
Maximum weight (metric tons)	24	30

Figure 4: ISO 668 container dimensions [2].

- **Open-top containers**. Hard-top containers without roof. Used for tall cargo.
- **Flatracks**. Flatracks consist of a floor structure with high loading capacity. Flatracks are used for heavy-lifts and overheight/overwidth cargo.
- **Platforms (Plats)**. Platforms consist solely of a floor structure and have extremely high loading capacity.
- **Ventilated containers**. Ventilated containers are passively ventilated. Often used for green coffee beans.
- **Insulated and refrigerated containers**. Insulated and refrigerated containers are mainly available as 20' and 40' containers. There are two subtypes.
  - Integral unit (integral reefer container). These containers carry their own refrigeration system and depends on power from the vessel or a diesel power generator (power-pack) fitted in the dimensions of a 20' container.
  - Porthole containers. These containers have no refrigeration system but rely on cold air provided by the ship. They are often called insulated containers.
- **Bulk containers**. Bulk containers have three loading hatches in the roof. They are mainly used to transport grains, foodstuffs, and spices.
- **Tank containers**. Tank containers are mainly used for foodstuffs and chemicals including hazardous materials.

In addition to high-cube containers, there are two other container types breaking the ISO standard. These are 45' containers (*Megabox* containers?) that are 45 foot long instead of 40, and *pallet-wide* (sea-cell) containers that are made a little wider to fit a standard pallet. They may be stacked with standard ISO containers, but may impose restrictions on the type of adjacent containers [4]. Other names for classifying containers includes *IMO* containers that contains explosives, radioactive material, or flammable material, Out-Of-Gate containers (*OOG*) have cargo sticking out (e.g., an open-top or flat-rack containers with over sized cargo), and reefer containers need external supply of power, cold air, and/or water (correct, or does reefer containers strictly refer to containers that require power?) [4]. There are a number of packing rules that must be met when stowing containers on the ship.

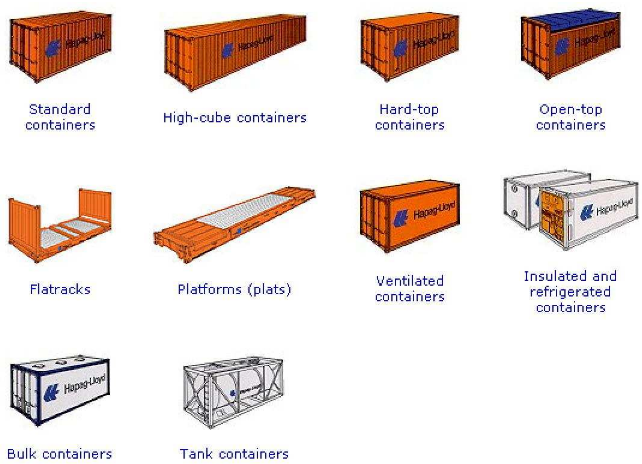


Figure 5: Major container types [9].

1. ISO stowage codes (e.g., infected cargo away from heat) (need precise rules...).
2. IMO rules (e.g., IMO containers may need to be a row or bay away from containers containing food) (need precise rules...).
3. Containers with military equipment may not be allowed in outer layers (is this an in-house rule?).
4. IMO sight line rules (e.g., containers cannot block the vision from the pilot house) (need precise rules...).
5. In-house rules.

4 Container Vessels

The cargo space of a container ship is made up of a number of *bays*, collections of stacks of containers along the length of the ship. Generally, each bay in the cargo space is divided into *over* deck and *under* deck by *hatch covers*, and sub-areas of a bay divided by hatch covers are called *holds*. Containers loaded under deck can be unloaded only after all containers loaded over deck on the hatch cover are removed as well as the hatch cover as shown in Figure 6. An example of the arrangement of holds for a bay with three hatch-covers is shown in Figure 7. Notice that in this example two intermediate holds exist that rest on two hatch-covers simultaneously. Each hold is composed of a group of *stacks*, and each stack is composed of vertically arranged groups of *cells*. Each cell is 20' long, 8' wide, and 8,6" high and is a physical location or a *slot* where a container is to be loaded [10, 11].

40' containers require two contiguous cells to be stored. Bays are counted from bow to stern and traditionally bays for 40' containers are given even numbers while bays for 20' containers are given odd numbers. A typical bay numbering scheme is shown to the right in Figure 6. Stacks are numbered from center and out as shown in the cross-section to the left in Figure 6. The level of a

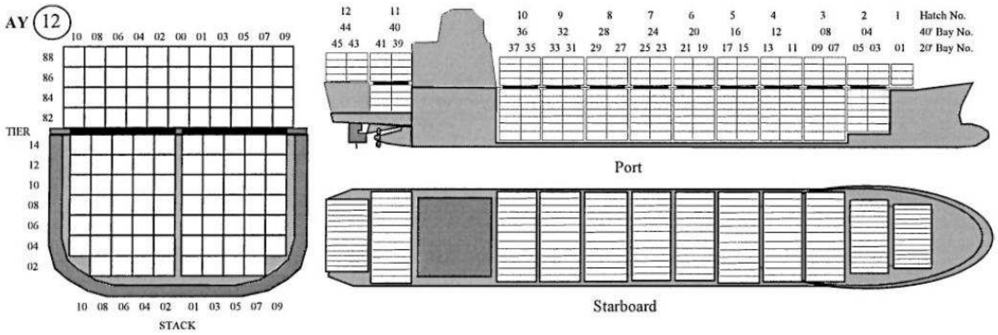


Figure 6: Stowage arrangement of a cellular container ship [11].

cell is given by its tier number. Tiers are counted from the bottom and up but traditionally starts at 80 above deck.

The number of hatches and the physical structure of bays depend on the location on the ship. Figure 8 shows the layout of a vessel. Notice that 20' containers are only allowed in certain cells on the ship and that special locations exist for reefer containers and cargo tainting odors.

No horizontal movement of containers is possible on the ship. Containers are loaded and unloaded vertically by quay cranes. Only free containers on top of stacks are accessible. On the other hand, stacks can be accessed independently of other stacks.

Each stack has maximum height and weight limits. The weight distribution of the ship must further ensure that the ship is stable and that weight stresses are within nominal limits. Stability is the tendency of a ship to return to its original position when disturbed after the disturbing force is removed. A ship becomes unstable if the vertical, transverse, or longitudinal distribution of the ship's weight is excessively unbalanced. The three most influential factors of a ships stability is *metacentric height* GM, heel and trim [3]. The metacentric height of a ship is defined as the vertical distance between the center of gravity G and the metacenter M of the ship as illustrated in

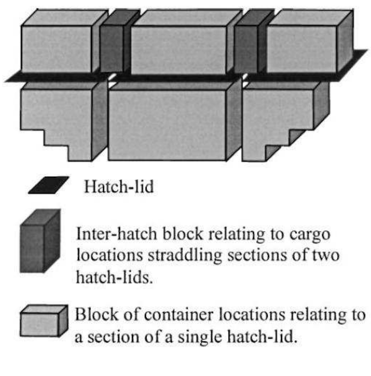


Figure 7: Example of cargo-space holds for a bay with three hatch-covers [11].

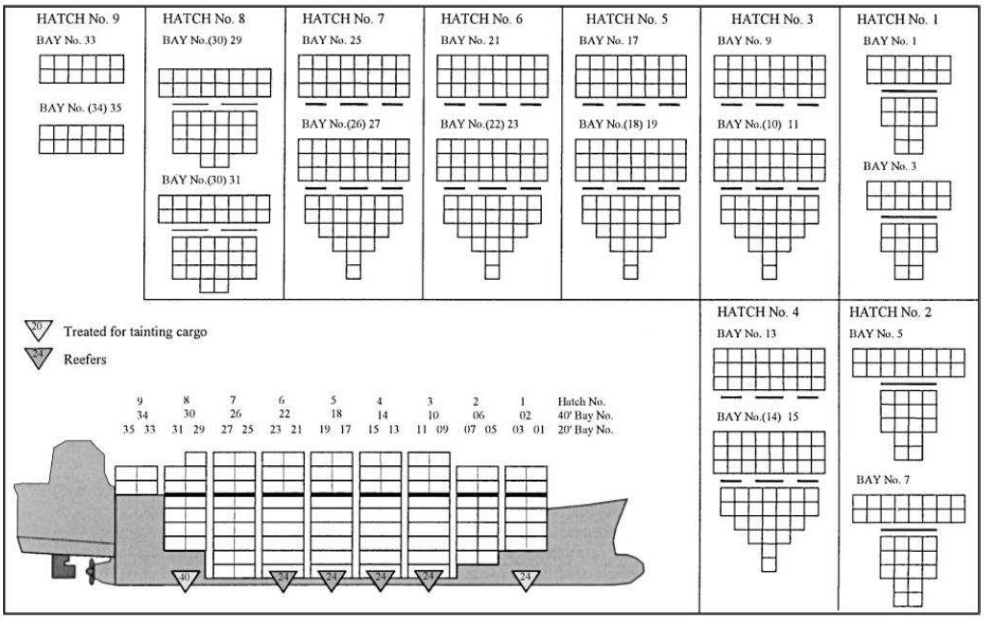


Figure 8: An example bay layout and distribution of 20', 40' and reefer cells [11].

Figure 9(a).[3] The heeling angle δ shown in Figure 9(a) is the angle between the ship's center line and vertical. The metacentric height and thus stability of the ship decrease as the absolute heeling angle increases. Nominal values of the heeling angle are close to zero (how close?). The trim angle θ is the longitudinal angle between vertical and the center line as shown in Figure 9(b). Similar to the heeling angle, the metacentric height in the longitudinal direction decreases as the absolute trim angle increases. Positive and negative trim angle is called trim by the stern and trim by the bow, respectively. A slight trim by the stern is a typical operational condition.

Excessive stress of the hull may also be caused by uneven weight distribution of cargo on the ship. Figure 10 illustrates stress caused by sagging, hogging, and torsion. Sagging and hogging is caused by too heavy cargo at the center or at the bow and stern of the ship, while torsion is caused by uneven transverse distribution of the cargo. A moderate uneven weight distribution can be corrected by filling ballast tanks under the stacks, but obviously the goal is to avoid the use of ballast water. The stability requirements when the ship is at berth are more relaxed since the water in the port basin is calm (but still it seems important e.g. to avoid strong heeling during unload and load, is this a practical problem that needs to be taken into account in the load plan?).

It is important to minimize the berth time. Not only to reduce port costs, but also to give slack time to deal with weather conditions like storms. Storms are avoided by all ships (but how strong weather conditions are within the nominal range of deep-sea container vessels?). Shorter

[3] The center of buoyancy B, is the centre of gravity of the volume of water which the hull displaces. The metacenter is the intersection of the vertical lines through the center of buoyancy of a floating body when it is at equilibrium and when it is floating at an angle due to a disturbance. The metacentric height is important because the righting force is proportional to the metacentric height times the sine of the angle δ of heel. A ship with a small GM will be "tender" - have a long roll period - and if it is too small will be at risk of capsizing in rough weather. A larger metacentric height on the other hand gives a ship a short roll period and good stability, but if it is too large the ship is "stiff" which may cause excessive stress of the hull in rough weather. Typical metracentric height is 1-2m.

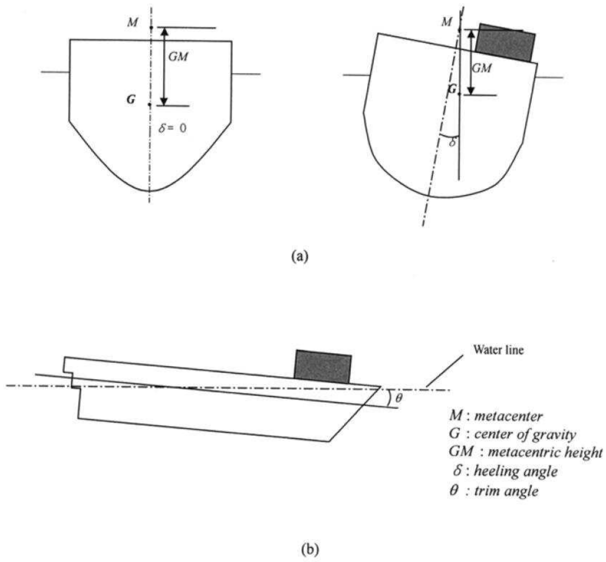


Figure 9: Stability of a ship. (a) GM and heel, (b) trim [3].

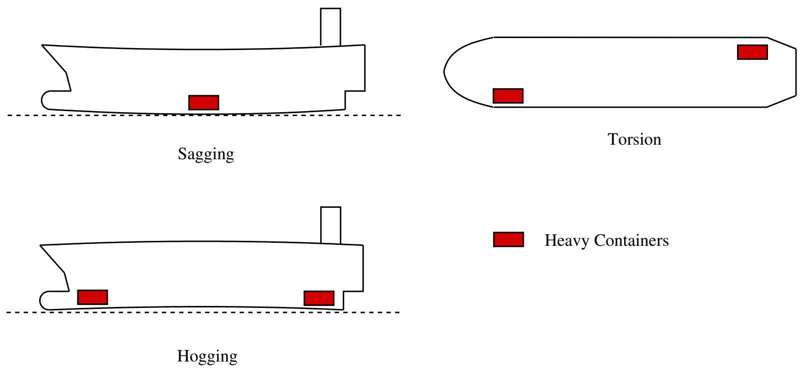


Figure 10: Stress of the hull caused by sagging, hogging, and torsion.

berth time also allows the ship to sail at lower speed to next port. This is important since the fuel consumption grows fast with speed (how fast?).

Short berth time is achieved by using the quay cranes efficiently (e.g., minimizing the make span of the quay cranes). This is hard to achieve since cranes drive on tracks and therefore cannot pass each other and since two cranes due to the distance between their legs are unable to work simultaneously on two adjacent bays.

5 Vessel State

A vessel is a static physical arrangement of 20' container cells. The dependency on hatch lids and quay cranes naturally partitions cells into holds and bays. In the remainder of the document, we use accents to partition a set E into elements over deck $\hat{E}$ and under deck $\check{E}$.

Definition 5.1 (Vessel) *A vessel $\mathcal{V}$ is a tuple $\mathcal{V} = \langle C, S, H, L, B \rangle$*[4]*, where*

- $C = \hat{C} \cup \check{C}$ is a set of 20' container cells. Each cell $c \in C$ is a pair $c \in \langle P_c, \vec{r}_c \rangle$ where $P_c \subseteq \{Pow, Wat, Air, Tai\}$ is a subset of properties indicating whether the cell supports power, water, cold air, and taint protection (other relevant properties?) and $\vec{r}_c$ is the physical location of the cell relative to an Euclidean coordinate system in SI units.

- $S = \hat{S} \cup \check{S}$ is a set of stacks. A stack is a column of slots. A slot consists of a single or two adjacent cells. Formally, a stack $s \in S$ is a pair $s = \langle C_s, h_s \rangle$, where $C_s \subseteq C$ is the partition of cells associated with the stack and h_s is the maximum height of the stack. Thus if the stack consists of single cells, we have $h_s = |C_s|$ and otherwise $2h_s = |C_s|$. The location of a cell c relative to a single stack is defined by indices $c_{j,k}$, where j is the height of the cell measured from the bottom of the stack and $k \in \{0, 1\}$ indicates whether c is towards the bow or stern of the ship in a double row stack.

- $H = \hat{H} \cup \hat{H}$ is a set of holds. Each hold $h \in H$ is a partition of stacks $S_h = \{s_1, \ldots, s_{|S_h|}\}$ enumerated from port to starboard. For convenience, C_h denotes the partition of cells of the stacks. The location of a cell c relative to a single hold is defined by indices $c_{i,j,k}$, where i is the stack number, j is the height of the cell measured from the bottom of the *lowest* stack, and $k \in \{0, 1\}$ as usual indicates whether c is towards the bow or stern of the ship in a double row stack.

- L is a set of hatch lids.

- $B = \{b_1, \ldots, b_{|B|}\}$ is a set of bays enumerated from bow to stern. A bay is defined as one or two *cell rows* that can be accessed simultaneously by a single quay crane. Cell rows are also enumerated from bow to stern. As shown in Figure 7, each bay consists of a set of holds and hatch lids. Formally, a bay $b \in B$ is a tuple $b = \langle T, H_b, L_b, d_b \rangle$, where T is an index set of cell rows, $H_b \subseteq H$ is a set of holds, $L_b \subseteq L$ is a set of hatch lids, and $d_b : H_b \to 2^{L_b}$ is a function that associates a hold with a set of hatch lids it depends on. For holds $\hat{h} \in \hat{H}$ over deck, $d(\hat{h})$ is the set of hatch lids that stacks of $\hat{h}$ rest on. For holds $\check{h} \in \check{H}$ under deck, $d_b(\check{h})$ is the set of hatch lids that must be removed in order to reach stacks of $\check{h}$.[5] For convenience, S_b and C_b denote the partition of stacks and cells of the holds. Since stacks often are vertically misaligned (e.g., see Figure 6 left), cells over and under deck are indexed independently. The position of a cell c relative to the partition of holds either over or under deck of a bay is given by indices $c_{i,j,k}$, where i is the stack number counted from port to starboard for all stacks in the holds, j is the tier height, and , and $k \in T$ is the cell row index of the cell. Figure 11 shows an indexing example.

A vessel state is an assignment of containers to cells. The vessel state is assumed only to change when the ship is being loaded or unloaded at berth at a container terminal.[6]

Definition 5.2 (Vessel State) *A vessel state is a tuple $\mathcal{S} = \langle \mathcal{V}, O, p \rangle$, where*

[4]This definition should include the size and location ballast tanks.
[5]If a bay b only stores containers above deck as the first bay shown in Figure 6, we have $\check{H}_b = \emptyset$ and $L_b = \emptyset$.
[6]If a ship has on board cranes this may not be the case.

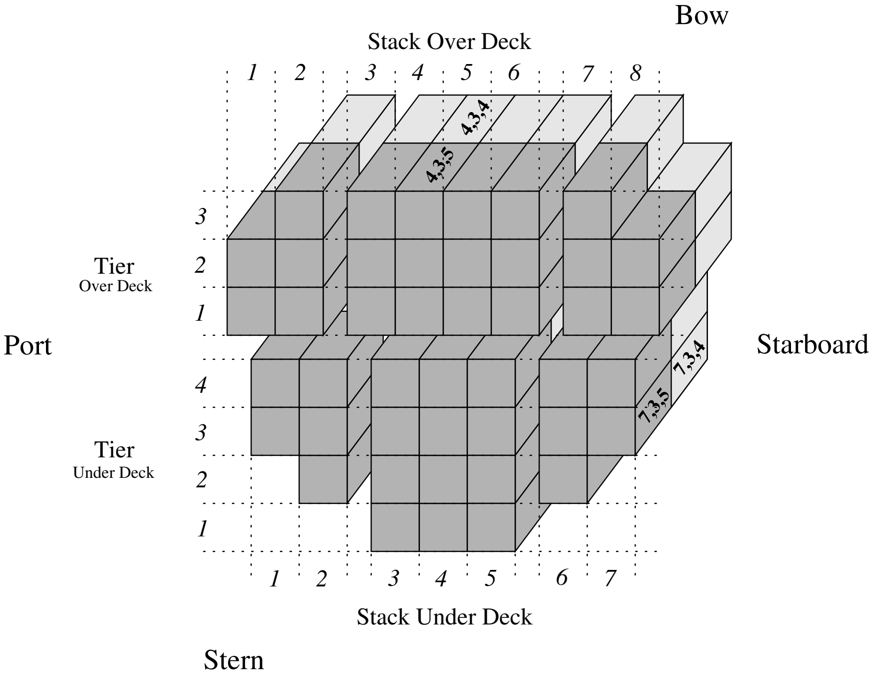


Figure 11: Example of indexing cells in a bay covering cell row 4 and 5.

- $\mathcal{V}$ is a vessel.
- O is a set of on board containers. Each container $o \in O$ is a tuple $o = \langle p_o, w_o, h_o, \vec{r}_o \rangle$, where $p_o \subseteq Co = \{Pow, Wat, Air, Tai, OOG, IMO, MIL, PW\}$ is a subset of properties defining whether o needs power, water, cold air, or is out-of-gate, IMO, with military content, or pallet-wide, w_o is the weight of o in kilograms, h_o is the height if o in meters, and $\vec{r}_o$ is the position of the center of gravity of the container.
- $p : O \to 2^C$ is a total function mapping containers to the set of cells they occupy.

6 Costs

The soft constraints of the container vessel stowage problem are defined by a cost function. To be comparable, costs are measured in a currency unit (dollars). Direct cost are port fees and other billed costs, while indirect costs cannot be directly measured or are preferences.

1. **Move cost**. A move cost is a fee paid for moving a container from the vessel to the yard or vice versa. A fixed price is paid no matter how the container is stored. In case of overstows for instance, the container may just rest shortly at the berth before being relocated, but still a full move cost is paid [5]. Actual move costs is in the range 30-200$ and varies a lot between ports [5]. Not all moves may be necessary to pay. Some percentage of the moves (e.g., 5%) may be free if these are shifting moves [5]. The move costs are reduced by minimizing the amount of shifting caused by overstows. Many models ignore the cost of moves that are not

caused by shifting, since the cost of these moves do not change no matter how the vessel is stowed. This, however, skews the relative size of the different cost components.

2. **Hatch lift cost**. The fee for of moving a hatch lid. Typically twice as much as the move cost [5] (correct?).

3. **Crane-set cost**. The fee for moving a quay crane along the ship to load/unload from a bay.

4. **Berth cost**. The fee for staying at berth at the container terminal and using the cranes for a particular period of time (are these one time fees plus hourly fees, or fees for use of each crane or both?). There are also indirect costs of long berth time that are hard to estimate. These include cost of extra speed to keep schedule, reduced flexibility with respect to weather conditions and other unforeseen changes (it may be best to summarize these indirect costs in a single parameter). The berth cost is minimized by minimizing the make span of the cranes. This is nontrivial since quay cranes depend on each other (cannot pass each other and must be separated by one bay) and schedules are defined in continuous time.

5. **Ballast water cost**. This is an indirect cost due to reduced profit.

6. **Suez cost**. Surcharge for stacks larger than 10 tiers at Suez (is this a fee for each single stack, is there a tolerance?).

7. **Preference costs** Among stowage plans with similar costs, some may be preferred (e.g, optional containers should be easy to reach). Such preferences can be achieved by small cost penalties (what are the most relevant preferences?)

7 Constraints

The hard constraints of the container vessel stowage problem are

1. All container packing rules described in Section 3 must be satisfied.

2. Containers must form physical stacks. A container cannot be allocated to a cell, if the cell below it is empty.

3. Containers must be placed in cells that provide what they need: water, cold air, power, taint protection etc.

4. OOG and open-top containers must be at specific locations (e.g., at the top of the stack)

5. Pallet wide containers may impose constraints on adjacent stacks. Due to lack of space, it may be necessary that containers adjacent to pallet wide containers are in ISO dimensions (need precise rules...)

6. Containers must be stowed such that twist-locks are accessible (need precise rules...).

7. 20' containers cannot be placed on top of 40' containers.

8. Wind stack force on outer stack must be within limits (need precise rules...).

9. Each stack must satisfy max weight and max height constraints. Max height constraints for stacks over deck may change depending on how much cargo is on board the ship. The heavier the ship is, the better the line-of-sight becomes and the higher the stacks over deck can be.

10. Stability constraints must be met: metacentric height, trim, heeling, and stress moments must be within limits.

8 Problem

Definition 8.1 (CVSP) *A container vessel stowage problem (CVSP) is a tuple* $CVSP = \langle s_0, p, LD, Q, f, R\rangle$*, where*

- s_0 is the initial state of the vessel.
- p is the number of ports.
- $LD = [ld_{i,j}]$ is a $p \times p$ load-discharge matrix, where $ld_{i,j}$ is a set of containers that must be loaded in port i and discharged in port j. Each container $l \in ld_{i,j}$ is a subset of properties $l \in Co$.
- $Q = \mathbb{N}^p$ defines the number of available quay cranes in the ports,
- $f = \{f_1, \dots, f_p\}$ is a set of cost functions $f_i : \mathcal{S}^2 \rightarrow \mathbb{R}$, where $f_i(s_{i-1}, s_i)$ defines the cost of changing the state of the vessel in port i.
- $R = \{r_1, \dots, r_p\}$ is a set of constraints $r_i : \mathcal{S}^2 \rightarrow \mathbb{B}$, where $r_i(s_{i-1}, s_i)$ defines the constraints in port i on the arrival and departure state (including that all containers are loaded and discharged as defined by LD).

Definition 8.2 (Valid Solution) *A valid solution to* $CVSP = \langle s_0, p, LD, Q, f, R\rangle$ *is a sequence of vessel states* $Sol = (s_1, \dots, s_p)$ *such that* $\bigwedge_{i=1}^{p} r_i(s_{i-1}, s_i)$.

Definition 8.3 (Optimal Solution) *An optimal solution to* $CVSP = \langle s_0, p, LD, Q, f, R\rangle$ *is a valid solution* $Sol^* = (s_1, \dots, s_p)$ *where* $\sum_{i=1}^{p} f_i(s_{i-1}, s_i)$ *is minimum.*

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