Software Programmable DSP Platform Analysis

Episode 6, Monday 12 March 2007

Dataflow Analysis

Reaching Definitions
Constant Propagation, Copy Propagation
Available Expressions, Reaching Expressions
Common Subexpression Elimination
Dead Code Elimination

Loop Optimizations

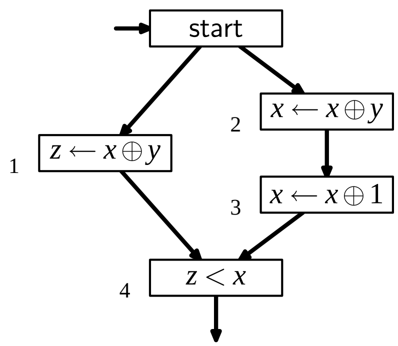
What is a loop? Loop Dominators.
Loop Invariants and Hoisting
Induction Variables. Strength Reduction
Loop Unrolling

Reaching Definitions

- An unambigous definition d of t is an assignment $t \leftarrow a \oplus b$ or $t \leftarrow M[a]$.
- A definition d reaches a statement u if there is a path of control edges leading from d to u that does not pass through any other definitions of t.

Reaching Definitions

Example



- Definition 1 reaches 4
- Definition 2 does **not** reach 4, because all paths from 2 to 4 path through 3 that kills 2.

Reaching Definitions: gen/kill sets

Defs(t): set of all definitions of temporary t.

statement s	$gen[s]$	$kill[s]$
$d : t \leftarrow b \oplus c$	$\{d\}$	$defs(t) - \{d\}$
$d : t \leftarrow M[b]$	$\{d\}$	$defs(t) - \{d\}$
$M[a] \leftarrow b$	$\{\}$	$\{\}$
if $a\ R\ b$ goto L_1 else goto L_2	$\{\}$	$\{\}$
goto L	$\{\}$	$\{\}$
$L :$	$\{\}$	$\{\}$
$f(a_1, \ldots, a_n)$	$\{\}$	$\{\}$
$d : t \leftarrow f(a_1, \ldots a_n)$	$\{d\}$	$defs(t) - \{d\}$

Calculating Reaching Definitions

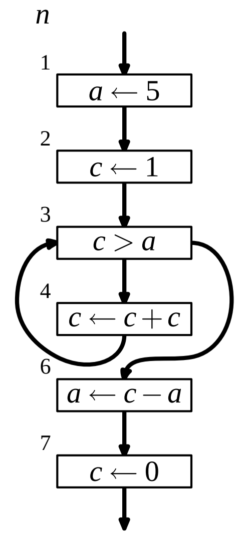
Initialize *in*[n] and *out*[n] to be empty sets.

Apply following equations until a fixpoint is reached:

$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

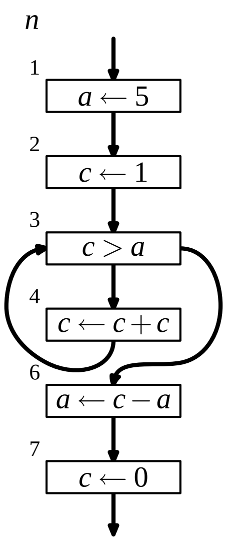
$$out[n] = gen[n] \cup (in[n] - kill[n])$$

Gen and *kill* sets are defined on previous slide.



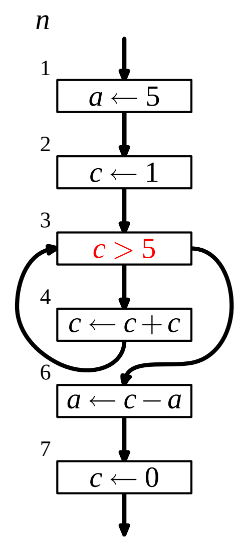
$a \leftarrow 5$
$c \leftarrow 1$
L_1 : if $c > 0$ goto L_2
$c \leftarrow c + c$
goto L_1
L_2 : $a \leftarrow c - a$
$c \leftarrow 0$

source: Appel Program 17.3 p.389



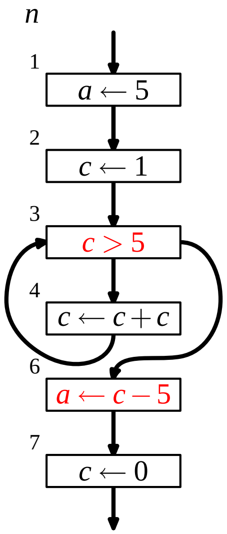
$gen[n]$	$kill[n]$	$in[n]$	$out[n]$
1	6	$\emptyset$	1
2	4, 7	1	1, 2
$\emptyset$	$\emptyset$	1, 2, 4	1, 2, 4
4	2, 7	1, 2, 4	1, 4
6	1	1, 2, 4	2, 4, 6
7	2, 4	2, 4, 6	6, 7

$in[n] = \bigcup_{p \in pred[n]} out[p]$ $\quad$ $out[n] = gen[n] \cup (in[n] - kill[n])$



$gen[n]$	$kill[n]$	$in[n]$	$out[n]$
1	6	$\emptyset$	1
2	4, 7	1	1, 2
$\emptyset$	$\emptyset$	1, 2, 4	1, 2, 4
4	2, 7	1, 2, 4	1, 4
6	1	1, 2, 4	2, 4, 6
7	2, 4	2, 4, 6	6, 7

The only def of a reaching 3 is 1, so can rewrite 3 to $c > 5$



$gen[n]$	$kill[n]$	$in[n]$	$out[n]$
1	6	$\emptyset$	1
2	4,7	1	1,2
$\emptyset$	$\emptyset$	1,2,4	1,2,4
4	2,7	1,2,4	1,4
6	1	1,2,4	2,4,6
7	2,4	2,4,6	6,7

Similarly with 6: $a \leftarrow c - 5$, but 4 cannot be rewritten.

Constant Propagation

- Let d be a statement: $t \leftarrow c$, where c is constant.
- Let n be another statement such as $y \leftarrow t \oplus x$.
- If d is the only definition of t reaching n,
- It is safe to rewrite n as $y \leftarrow c \oplus x$.

Copy Propagation

- Copy propagation is like constant propagation, but instead of constant c a variable is used.
- Let $d : t \leftarrow z$ be a statement.
- Let $n : y \leftarrow t \oplus x$ be a statement using t.
- If d is the only definition of t reaching n and there is no definition of z on **any** path from d to n then we can rewrite: $n : y \leftarrow z \oplus x$.
- This may remove t entirely from the program.
- Mind the "any" requirement: this includes paths that cross n more than once (for example loops), so the redefinition after n can also prevent copy propagation.

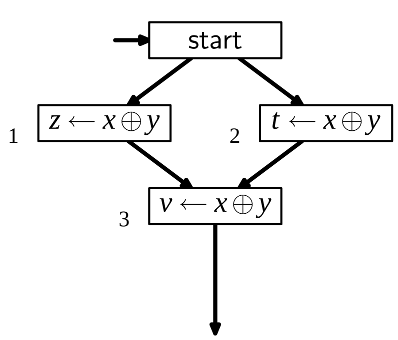
Available Expressions

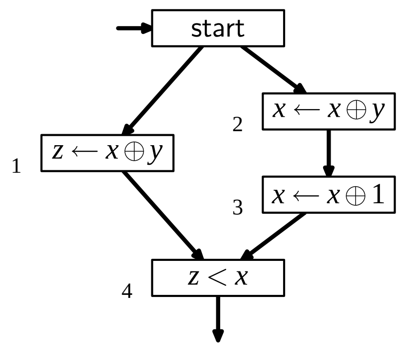
An expression $x \oplus y$ is available at a node n in the flow graph if:

- on every path from the entry node to n, $x \oplus y$ is computed at least once,
- and there are no definitions of x or y since the most recent occurrence of $x \oplus y$ on that path.

Available Expressions

Example





$x \oplus y$ is available in 3 $\qquad$ $x \oplus y$ is not available in 4

Computing Available Expressions

statement s	$gen[s]$	$kill[s]$
$d : t \leftarrow b \oplus c$	$\{b \oplus c\} - kill[s]$	all containing t
$d : t \leftarrow M[b]$	$\{M[b] - kill[s]\}$	all containing t
$M[a] \leftarrow b$	$\{\}$	all $M[x]$
if $a\ R\ b$ goto L_1 else goto L_2	$\{\}$	$\{\}$

$$in[n] = \bigcap_{p \in pred[n]} out[p] \quad \text{if } n \text{ is not entry}$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

Initialize $in[\text{entry}]$ to empty set, initialize all other sets to contain all expressions of the program. Iterate until (the greatest) fixpoint is reached.

Reaching Expressions

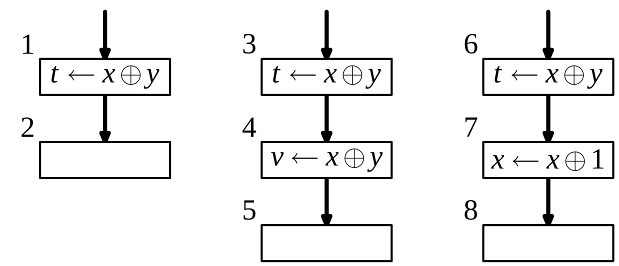
Reaching expressions, are much like reaching definitions. Expression $t \leftarrow x \oplus y$ in node s reaches a node n if:

- there is a path from s to n that
- does not go through any assignment to x or y,
- or through any other computation of $x \oplus y$.

Reaching expressions are characterized by their own *gen*, *kill* and *in*, *out* equations as for previous flow analyses. They are computed very much like previous examples.

Reaching Expressions

Example



- $1 : x \oplus y$ is reaching 2.
- $3 : x \oplus y$ is not reaching 5, as 4 recomputes it.
- But 4 is reaching 5.
- $6 : x \oplus y$ is not reaching 8, as 7 kills x.

Common Subexpression Elimination

If expression $x \oplus y$ is **available** at $s : t \leftarrow x \oplus y$ then the computation of $x \oplus y$ within s can be eliminated:

- Compute expressions $x \oplus y$ **reaching** s.
- Introduce a new (fresh) temporary w.
- For each such reaching node $n : v \leftarrow x \oplus y$ rewrite n to be:

$$n : w \leftarrow x \oplus y$$
$$n' : v \leftarrow w$$

- Modify s to use w: $s : t \leftarrow w$

Common Subexpression Elimination
Example

```
x = a + b + c;
y = a + b + d;
```

compiles to	CSE	copy prop.	reg. alloc.
$x \leftarrow a+b$	$w \leftarrow a+b$	$w \leftarrow a+b$	$y \leftarrow a+b$
$x \leftarrow x+c$	$x \leftarrow w$	$x \leftarrow w+c$	$x \leftarrow y+c$
$y \leftarrow a+b$	$x \leftarrow x+c$	$y \leftarrow w+d$	$y \leftarrow y+d$
$y \leftarrow y+d$	$y \leftarrow w$		
	$y \leftarrow y+d$		

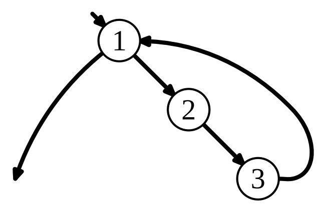
Dead Code Elimination

If $s : a \leftarrow b \oplus c$ (or $s : a \leftarrow M[x]$) and a is **not** live-out of s then the instruction can be eliminated.

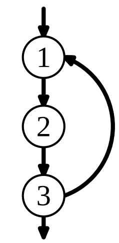
```
x = a + b + c;
y = a + b + d;
return y;
```

compiles to	*live-in*[s]	*live-out*[s]	DCE
$x \leftarrow a+b$	a,b	a,b,c,x	$x \leftarrow a+b$
$x \leftarrow x+c$	a,b,c,x	a,b	
$y \leftarrow a+b$	a,b	d,y	$y \leftarrow a+b$
$y \leftarrow y+d$	d,y	y	$y \leftarrow y+d$

Loops
Examples



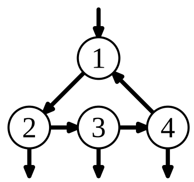
a *while-do* loop

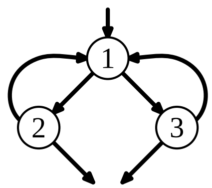


a *do-while* loop
(also known as *repeat-until*)

Loops with Multiple Exit Points

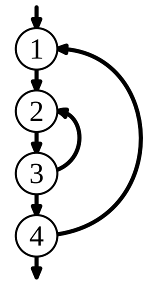
Examples

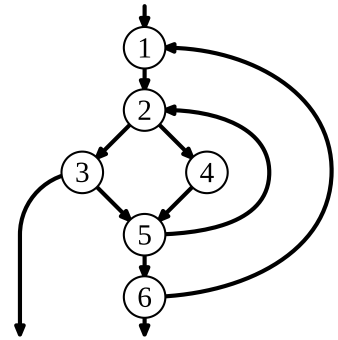




Nested Loops

Examples





nested “do-while” loops nested loops with “break”

Many loop structures cry for an abstract definition.

Loops Precisely Defined

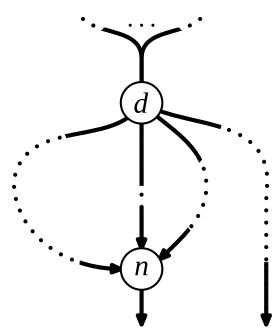
A set of nodes S constitutes a loop if:

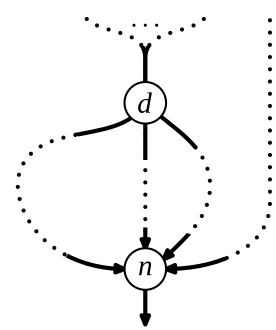
- S contains a header node h such that
- from any node in S there is a path leading to h.
- There are not any edges from nodes outside S to nodes in S other than h.

All loops on previous slides are loops according to this definition.

Loop Dominator

Node d dominates node n if every path of directed edges from s_0 to n must go through d. Every node dominates itself.





d dominates n d does not dominate n

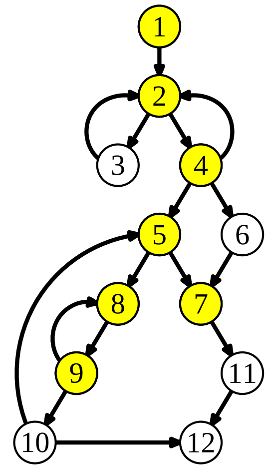
Computing Dominators

Dominators are computed by iterating the following equations over the nodes of the flow graph:

$$D[s_0] = \{s_0\}$$
$$D[n] = \{n\} \cup \Big(\bigcap_{p \in pred[n]} D[p] \Big) \text{ for } n \neq s_0$$

Initially each $D[n]$ should contain all nodes of the graph (except $D[n_0]$).

Computing Dominators
Example



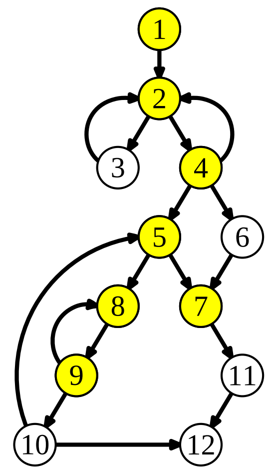
$D[1]$	1	immediate dominators
$D[2]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[3]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[4]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[5]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[6]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[7]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[8]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[9]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[10]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[11]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	
$D[12]$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	

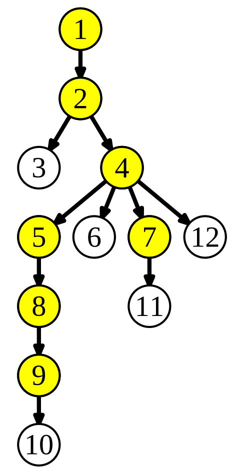
Dominator Tree

Every node n has at most one *immediate dominator* $idom[n]$ such that:

- $idom(n)$ is not the same node as n.
- $idom(n)$ dominates n.
- $idom(n)$ does not dominate any other dominator of n.

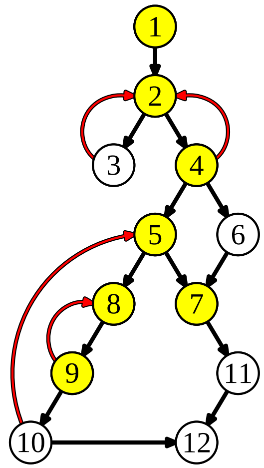
Dominator Tree
Example





Back Edge

An edge from node n to h, where h dominates n.

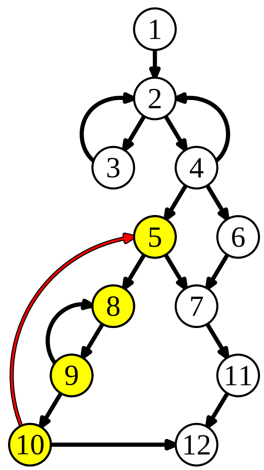


Loop Definition Revisitted

- The natural loop of a back-edge $n \rightarrow h$ is the set of nodes x, such that h dominates x and there is a path from x to n not containing h.
- Node h is the header of the loop.
- This definition allows automatic detection of loops.

Natural Loop

Example



- Natural loop of back edge $10 \rightarrow 5$
- Includes nodes: $5, 8, 9, 10$
- Contains the loop $8, 9$ nested.

Loop Invariant

The definition $d : t \leftarrow a_1 \oplus a_2$ is a loop invariant within loop L if $d \in L$ and for each operand a_i:

- a_i is constant,
- or all the definitions of a_i reaching d are outside the loop,
- or only one definition of a_i reaches d and that definition is loop invariant.

Loop invariant computations can sometimes be moved out (*hoisted*) out of the loop, speeding up the execution.

Can We Hoist $t \rightarrow a \oplus b$?

$L_0 : t \leftarrow 0$
$L_1 : i \leftarrow i + 1$
$\quad t \leftarrow a \oplus b$
$\quad M[i] \leftarrow t$
$\quad \text{if } i < N \text{ goto } L_1$
$L_2 : x \leftarrow t$

- $t \leftarrow a \oplus b$ is loop invariant.
- Moving it before the loop would not change the behaviour of our program.
- It would make the program faster.
- So the answer is: **YES!**

Hoisted.

$L_0 : t \leftarrow 0$
$\quad t \leftarrow a \oplus b$
$L_1 : i \leftarrow i + 1$
$\quad M[i] \leftarrow t$
$\quad \text{if } i < N \text{ goto } L_1$
$L_2 : x \leftarrow t$

- $t \leftarrow a \oplus b$ is loop invariant.
- Moving it before the loop would not change the behaviour of our program.
- It would make the program faster.
- So the answer is: **YES!**

Can We Hoist $t \rightarrow a \oplus b$?

$L_0 : t \leftarrow 0$
$L_1 : \text{if } i \geq N \text{ goto } L_2$
$\quad i \leftarrow i + 1$
$\quad t \leftarrow a \oplus b$
$\quad M[i] \leftarrow t$
$\quad \text{goto } L_1$
$L_2 : x \leftarrow t$

- The original program does not always execute $t \leftarrow a \oplus b$.
- Hoisting would execute it unconditionally always at least once.
- Leading to a wrong value of x if no loop iterations are executed.
- So the answer is: **NO!**

Minimum trip count pragma might help though...

Can We Hoist $t \rightarrow a \oplus b$?

$L_0 : t \leftarrow 0$
$L_1 : i \leftarrow i + 1$
$\quad t \leftarrow a \oplus b$
$\quad M[i] \leftarrow t$
$\quad t \leftarrow 0$
$\quad M[j] \leftarrow t$
$\quad \text{if } i < N \text{ goto } L_1$
$L_2 :$

- The original program has more than one def of t.
- Hoisting would change the interleaving of the assignments.
- So the answer is: **NO!**

Can We Hoist $t \rightarrow a \oplus b$?

$$
\begin{aligned}
L_0 :& t \leftarrow 0 \\
L_1 :& M[j] \leftarrow t \\
& i \leftarrow i+1 \\
& t \leftarrow a \oplus b \\
& M[i] \leftarrow t \\
& \text{if } i < N \text{ goto } L_1 \\
L_2 :& x \leftarrow t
\end{aligned}
$$

- t is used before the loop invariant definition.
- So the answer is: **NO!**

Sufficient Conditions for Hoisting

Loop invariant computation $d : t \leftarrow a \oplus b$ can be hoisted if:

- d dominates all loop exits at which t is live-out.
- There is only one def of t in the loop.
- t is not live-out of the loop preheader.

Basic Induction Variable

$$
\begin{aligned}
& s \leftarrow 0 \\
& i \leftarrow 0 \\
L_1 :& \text{if } i \geq n \text{ goto } L_2 \\
& j \leftarrow i \cdot 4 \\
& k \leftarrow j + a \\
& x \leftarrow M[k] \\
& s \leftarrow s + x \\
& i \leftarrow i + 1 \\
& \text{goto } L_1 \\
L_2 :&
\end{aligned}
$$

The variable i is a basic induction variable in a loop L with header node h if the only definitions of i within L are of the form $i \leftarrow i + c$ or $i \leftarrow i - c$, where c is loop invariant.

Derived Induction Variable

$$
\begin{aligned}
& s \leftarrow 0 \\
& i \leftarrow 0 \\
L_1 :& \text{if } i \geq n \text{ goto } L_2 \\
& j \leftarrow i \cdot 4 \\
& k \leftarrow j + a \\
& x \leftarrow M[k] \\
& s \leftarrow s + x \\
& i \leftarrow i + 1 \\
& \text{goto } L_1 \\
L_2 :&
\end{aligned}
$$

k is a derived induction variable if L contains only one definition of k, $k \leftarrow j \cdot c$ or $k \leftarrow j + d$, where j is an induction variable and c, d are invariant.

If j is an induction variable derived from i then the only def of j that reaches k is the one in the loop, and there is no def of i between the def of j and the def of k.

Strength Reduction

- On many machines multiplication is more expensive than addition (including C67xx).
- a definition of derived variable like $j \leftarrow i \cdot c$ can be replaced with addition.

$$
\begin{array}{l}
\quad s \leftarrow 0 \\
\quad i \leftarrow 0 \\
L_1 : \text{if } i \geq n \text{ goto } L_2 \\
\quad j \leftarrow i \cdot 4 \\
\quad k \leftarrow j + a \\
\quad x \leftarrow M[k] \\
\quad s \leftarrow s + x \\
\quad i \leftarrow i + 1 \\
\quad \text{goto } L_1 \\
L_2 :
\end{array}
\qquad
\begin{array}{l}
\quad s \leftarrow 0 \\
\quad i \leftarrow 0 \\
\quad j' \leftarrow 0 \;\; k' \leftarrow a \\
L_1 : \text{if } i \geq n \text{ goto } L_2 \\
\quad j \leftarrow j' \\
\quad k \leftarrow k' \\
\quad x \leftarrow M[k] \\
\quad s \leftarrow s + x \\
\quad i \leftarrow i + 1 \;\; j' \leftarrow j' + 4 \;\; k' \leftarrow k' + 4 \\
\quad \text{goto } L_1 \\
L_2 :
\end{array}
$$

Dead code elimination will remove $j \leftarrow j'$.
Elimination of useless variables (Appel p.424) eliminates $j' \leftarrow j' + 4$ too.

Loop Unrolling

Some loops have such a small body that most of the time is spent incrementing the loop counter variable and testing the loop-exit condition.

We can make these loops more efficient by unrolling them, putting two or more copies of the loop body in a row.

Let loop L have header h and back edges $s : s_i \rightarrow h$.
We unroll L as follows:

- Copy the nodes to make a loop L' with header h' and back edges $s_i' \rightarrow h'$.
- Change all the back edges in L from $s_i \rightarrow h$ to $s_i \rightarrow h'$.
- Change all the back edges in L' from $s_i' \rightarrow h'$ to $s_i' \rightarrow h$.

Useless Loop Unrolling
Example

$$
\begin{array}{ll}
L_1: & x \leftarrow M[i] \\
& s \leftarrow s + x \\
& i \leftarrow i + 4 \\
& \text{if } i < n \text{ goto } L_1 \text{ else } L_2 \\
L_2: &
\end{array}
$$

$$
\begin{array}{ll}
L_1: & x \leftarrow M[i] \\
& s \leftarrow s + x \\
& i \leftarrow i + 4 \\
& \text{if } i < n \text{ goto } L_1' \text{ else } L_2 \\
L_1': & x \leftarrow M[i] \\
& s \leftarrow s + x \\
& i \leftarrow i + 4 \\
& \text{if } i < n \text{ goto } L_1 \text{ else } L_2
\end{array}
$$

Useful Loop Unrolling
Example

Use information about induction vars to combine increments. This works for even number of iterations:

$$
\begin{array}{ll}
L_1: & x \leftarrow M[i] \\
& s \leftarrow s + x \\
& i \leftarrow i + 4 \\
& \text{if } i < n \text{ goto } L_1 \text{ else } L_2 \\
L_2: &
\end{array}
$$

$$
\begin{array}{ll}
L_1: & x \leftarrow M[i] \\
& s \leftarrow s + x \\
& x \leftarrow M[i+4] \\
& s \leftarrow s + x \\
& i \leftarrow i + 8 \\
& \text{if } i < n \text{ goto } L_1 \text{ else } L_2 \\
L_2: &
\end{array}
$$

General version in Appel p.430.

Some Optimizations of `cl6x`

- -O0 register allocation, loop rotation, dead code elimination, keyword driven inlining
- -O1 copy/constant propagation, useless variable elimination, common subexpression elimination
- -O2 software pipelining, loop optimizations, global common subexpression elimination, global useless variable elimination, strength reduction with arrays and pointers, loop unrolling,
- -O3 unsued function elimination, automatic inlining, (limited) partial evaluation,

We have now covered most of these optimizations!