Algorithms and Data Structures, Fall 2011

Stacks and queues

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Based on slides by Kevin Wayne, Princeton

Fundamental data types.

- Values: sets of objects.
- Operations: insert, remove, test if empty.
- Intent is clear when we insert.
- Which item do we remove?

LIFO = "last in first out"

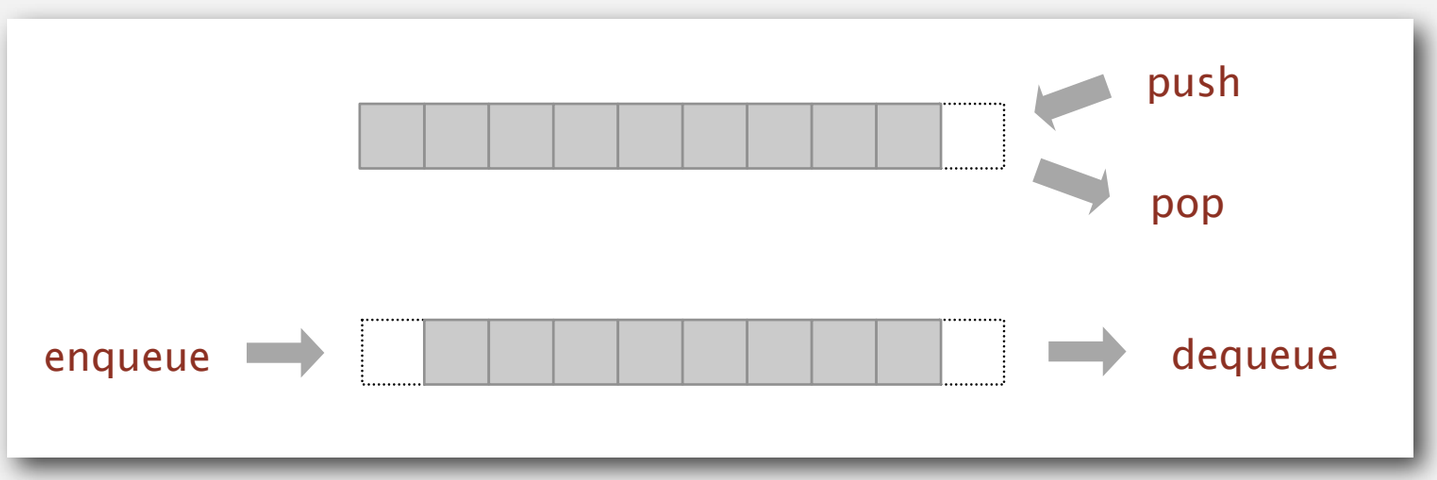
Stack. Remove the item most recently added.

Analogy. Cafeteria trays, Web surfing.

FIFO = "first in first out"

Queue. Remove the item least recently added.

Analogy. Line in the canteen.



Client, implementation, interface

Separate interface and implementation.

Ex: stack, queue, priority queue, symbol table, union-find, ...

Benefits.

- Client can't know details of implementation $\Rightarrow$
  client has many implementation from which to choose.
- Implementation can't know details of client needs $\Rightarrow$
  many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

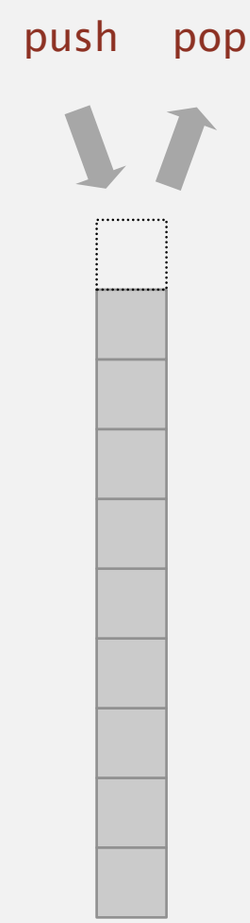
Client: program using operations defined in interface.
Implementation: actual code implementing operations.
Interface: description of data type, basic operations.

- stacks
- resizing arrays
- queues
- generics
- iterators
- applications

Stack API

Warmup. Stack of strings objects.

public class StackOfStrings	
StackOfStrings()	*create an empty stack*
void push(String s)	*insert a new item onto stack*
String pop()	*remove and return the item most recently added*
boolean isEmpty()	*is the stack empty?*
int size()	*number of items on the stack*



Challenge. Reverse sequence of strings from standard input.

Stack: linked-list implementation in Java

```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      ...
   }

   public String pop()
   {
      ...
   }
}
```

inner class

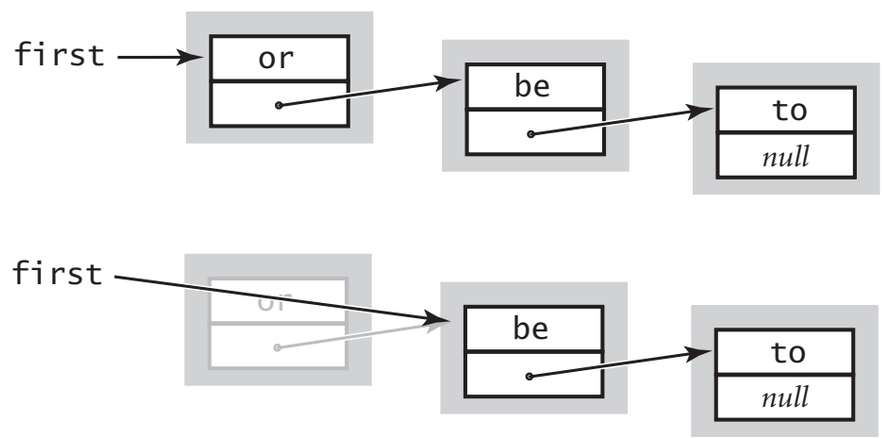
Stack pop: linked-list implementation

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



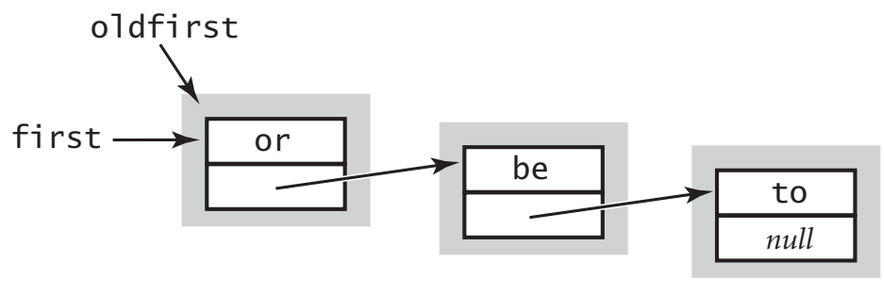
return saved item

```
return item;
```

Stack push: linked-list implementation

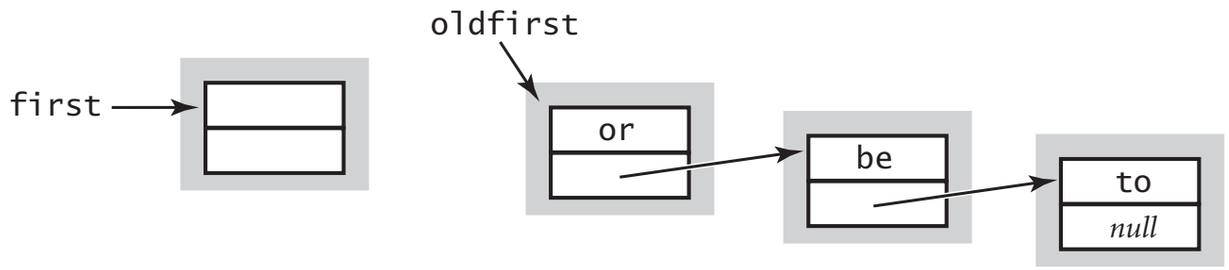
save a link to the list

```
Node oldfirst = first;
```



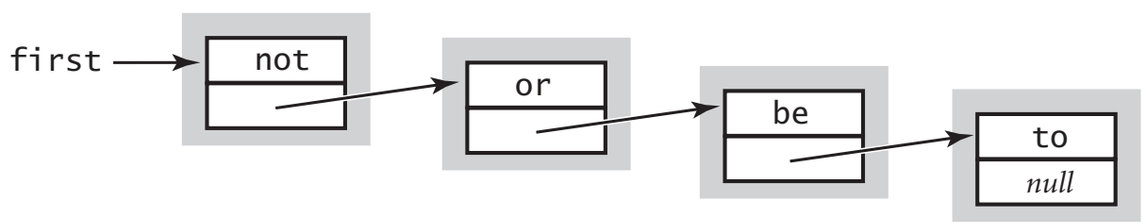
create a new node for the beginning

```
first = new Node();
```



set the instance variables in the new node

```
first.item = "not";
first.next = oldfirst;
```



Stack: linked-list implementation in Java

```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

Stack: array implementation

Array implementation of a stack.

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.

`s[]`	to	be	or	not	to	be	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

`N` `capacity = 10`

Defect. Stack overflows when `N` exceeds capacity. [stay tuned]

Stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

a cheat (stay tuned)

decrement N;
then use to index into array

Stack considerations

Overflow and underflow.

- Underflow: throw exception if pop from an empty stack.
- Overflow: use resizing array for array implementation. [stay tuned]

Loitering. Holding a reference to an object when it is no longer needed.

```
public String pop()
{   return s[--N];   }
```

loitering

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   return item;
}
```

this version avoids "loitering":
garbage collector reclaims memory
only if no outstanding references

Null items. We allow null items to be inserted.

- stacks
- **resizing arrays**
- queues
- generics
- iterators
- applications

Stack: dynamic-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

Problem session: Think about this!

(And try not to look at the next few slides.)

Stack: dynamic-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

First try.

- `push()`: increase size of `s[]` by 1.
- `pop()`: decrease size of `s[]` by 1.

Too expensive.

- Need to copy all item to a new array.
- Inserting first N items takes time proportional to $1 + 2 + \ldots + N \sim N^2 / 2$.

infeasible for large N

Challenge. Ensure that array resizing happens infrequently.

Stack: dynamic-array implementation

"repeated doubling"

Q. How to grow array?

A. If array is full, create a new array of twice the size, and copy items.

```
public StackOfStrings() {  s = new String[1];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

cost of array resizing is now

$2 + 4 + 8 + \ldots + N \sim 2N$

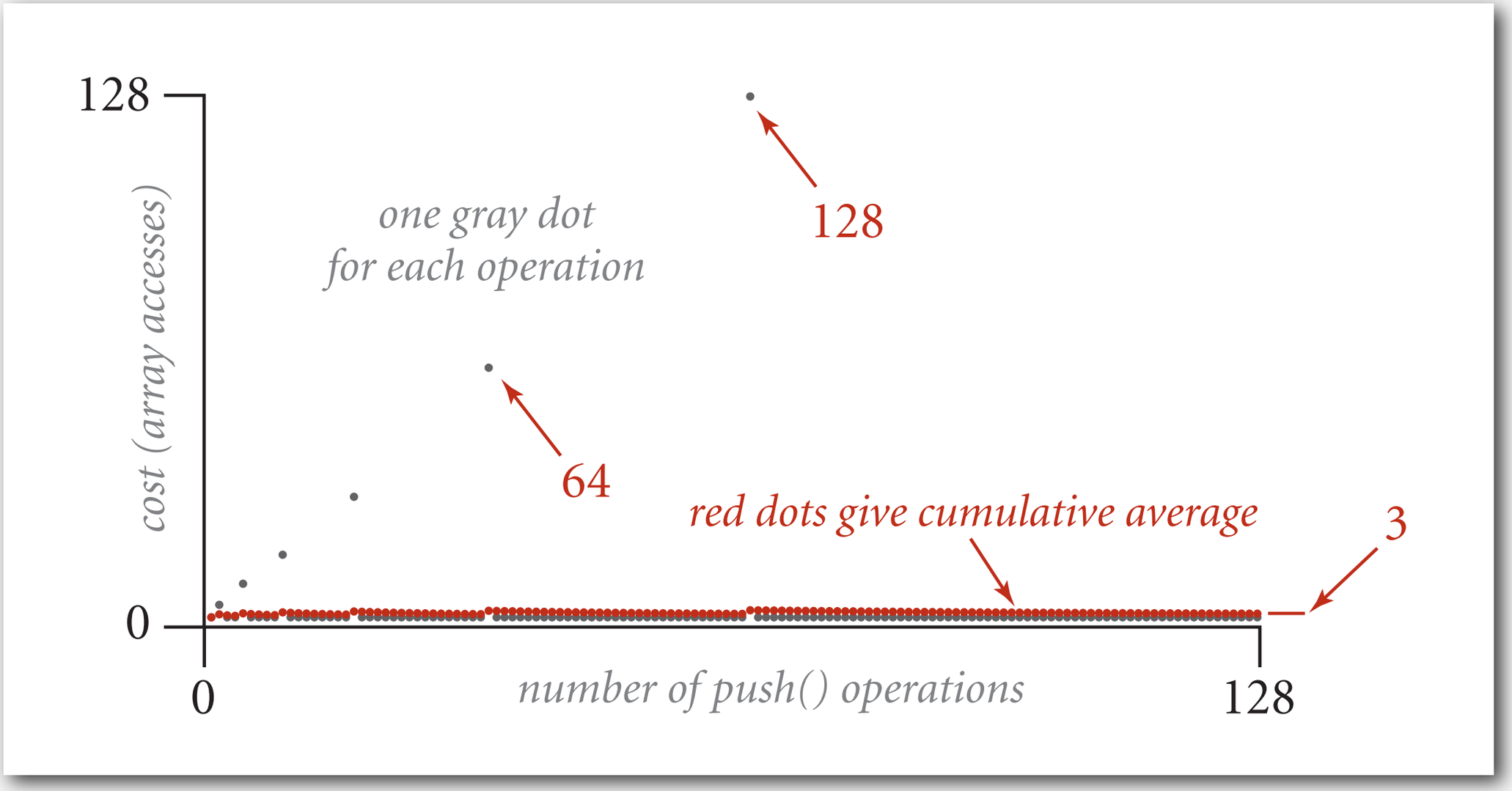
Consequence. Inserting first N items takes time proportional to N (not N^2).

Stack: amortized cost of adding to a stack

Cost of inserting first N items. $N + (2 + 4 + 8 + \ldots + N) \sim 3N.$

1 array accesses per push

k array accesses to double to size k



Stack: dynamic-array implementation

Q. How to shrink array?

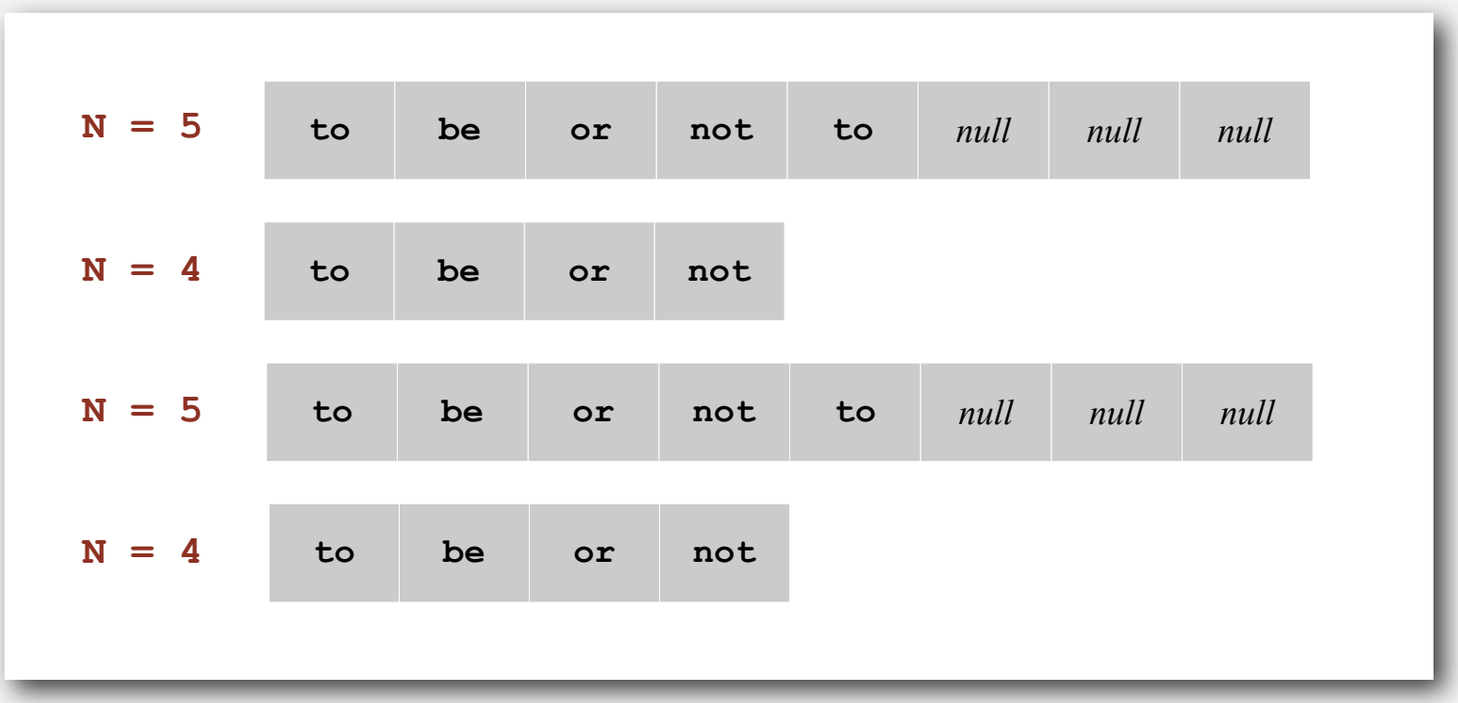
First try.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is one-half full.

Too expensive.

"thrashing"

- Consider push-pop-push-pop-... sequence when array is full.
- Takes time proportional to N per operation in worst case.



Stack: dynamic-array implementation

Q. How to shrink array?

Efficient solution.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length / 2);
   return item;
}
```

Invariant. Array is between 25% and 100% full.

Stack dynamic-array implementation: performance

Amortized analysis. Average running time per operation over a worst-case sequence of operations.

Proposition. Starting from empty stack (with dynamic resizing), any sequence of M push and pop operations takes time proportional to M.

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and shrinking

running time for doubling stack with N items

Stack implementations: dynamic array vs. linked List

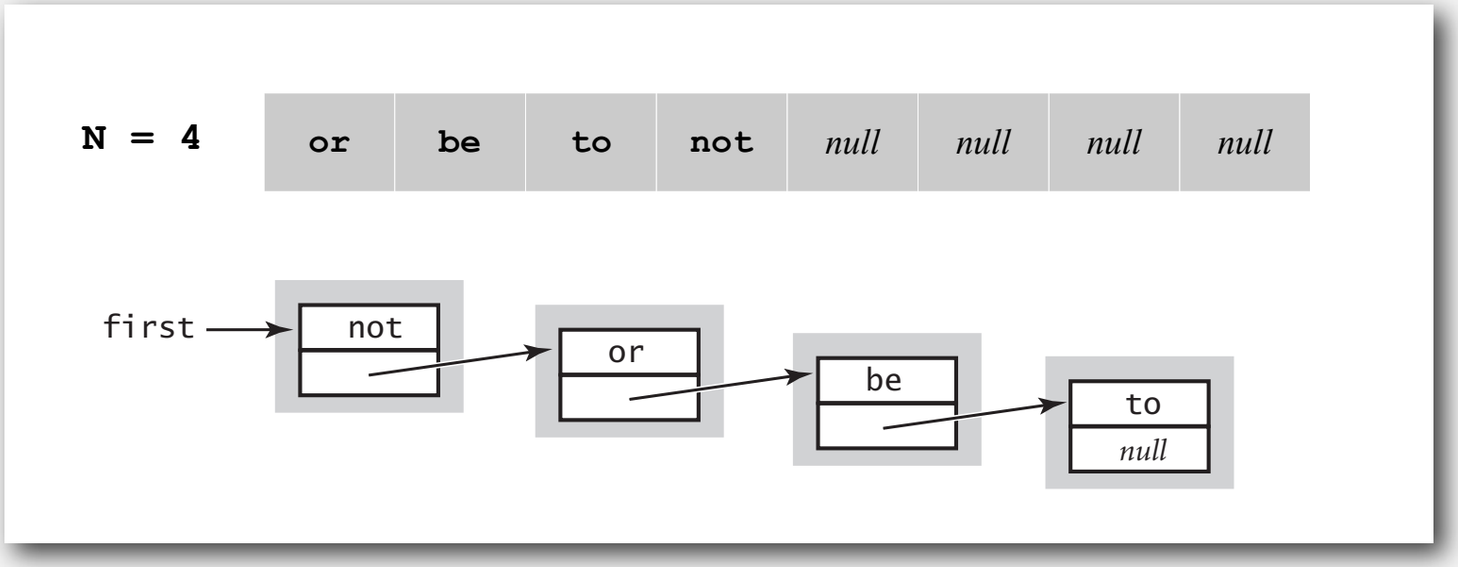
Tradeoffs. Can implement a stack with either dynamic array or linked list; client can use interchangeably. Which one is better?

Linked-list implementation.

- Every operation takes constant time in the worst case.
- Uses extra time and space to deal with the links.

Dynamic-array implementation.

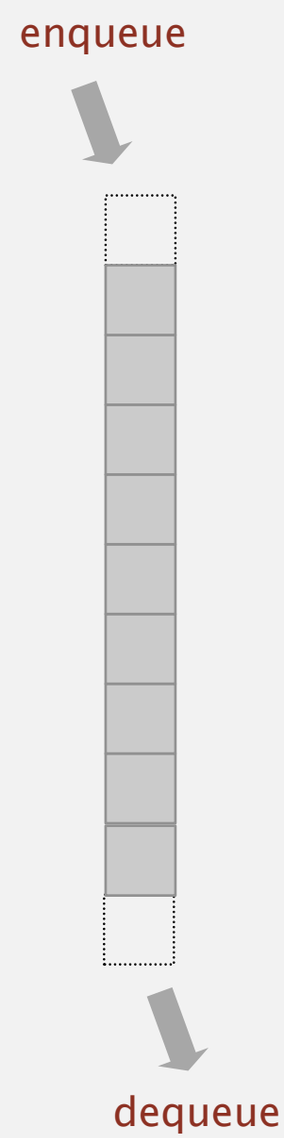
- Every operation takes constant amortized time.
- Less wasted space.



- stacks
- resizing arrays
- queues
- generics
- iterators
- applications

Queue API

public class QueueOfStrings	
QueueOfStrings()	*create an empty queue*
void enqueue(String s)	*insert a new item onto queue*
String dequeue()	*remove and return the item least recently added*
boolean isEmpty()	*is the queue empty?*
int size()	*number of items on the queue*



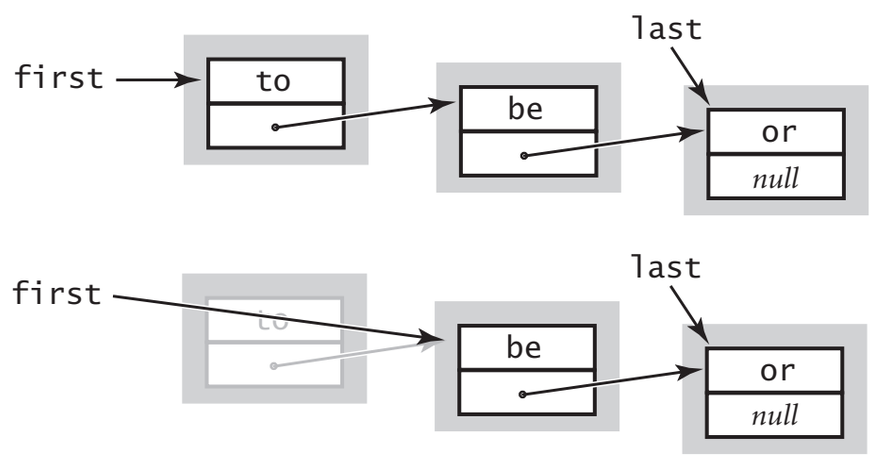
Queue dequeue: linked-list implementation

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



return saved item

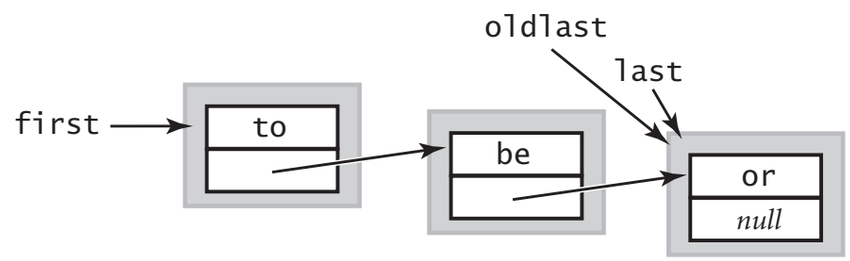
```
return item;
```

Remark. Identical code to linked-list stack `pop()`.

Queue enqueue: linked-list implementation

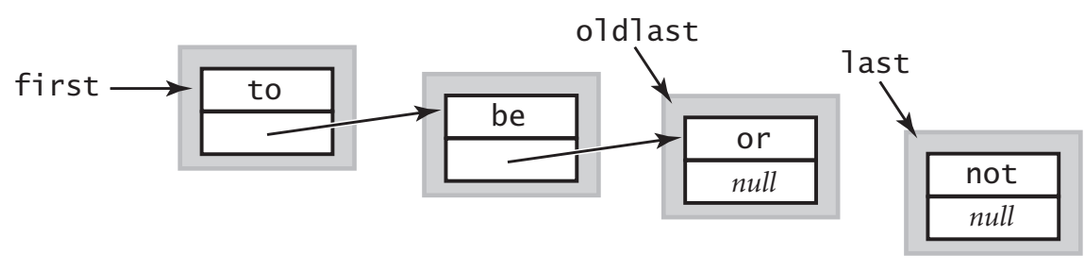
save a link to the last node

```
Node oldlast = last;
```



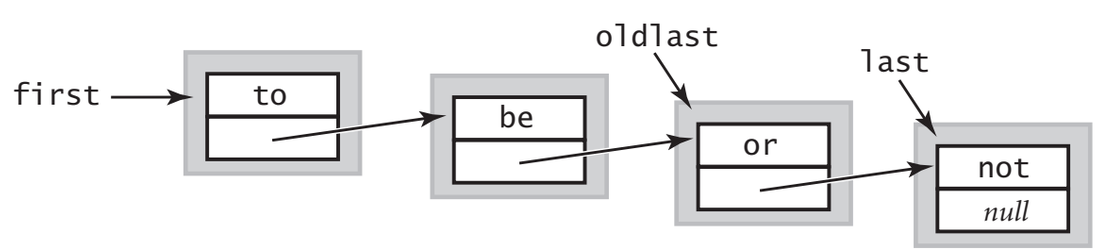
create a new node for the end

```
Node last = new Node();
last.item = "not";
last.next = null;
```



link the new node to the end of the list

```
oldlast.next = last;
```



Queue: linked-list implementation in Java

```
public class QueueOfStrings
{
   private Node first, last;

   private class Node
   {  /* same as in StackOfStrings */  }

   public boolean isEmpty()
   {  return first == null;  }

   public void enqueue(String item)
   {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```

special cases for empty queue

Queue: dynamic array implementation

Array implementation of a queue.

- Use array `q[]` to store items in queue.
- `enqueue()`: add new item at `q[tail]`.
- `dequeue()`: remove item from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.
- Add dynamic resizing.

q[]	*null*	*null*	**the**	**best**	**of**	**times**	*null*	*null*	*null*	*null*
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

head **tail** **capacity = 10**

- stacks
- resizing arrays
- queues
- generics
- iterators
- applications

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

Attempt 1. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

@#$*! most reasonable approach until Java 1.5.

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

Attempt 2. Implement a stack with items of type `Object`.

- Casting is required in client.
- Casting is error-prone: run-time error if types mismatch.

```
StackOfObjects s = new StackOfObjects();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfVans`, etc.?

Attempt 3. Java generics.

- Avoid casting in client.
- Discover type mismatch errors at compile-time instead of run-time.

type parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = s.pop();
```

compile-time error

Guiding principles. Welcome compile-time errors; avoid run-time errors.

Generic stack: linked-list implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

generic type name

- stacks
- resizing arrays
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- **iterators**
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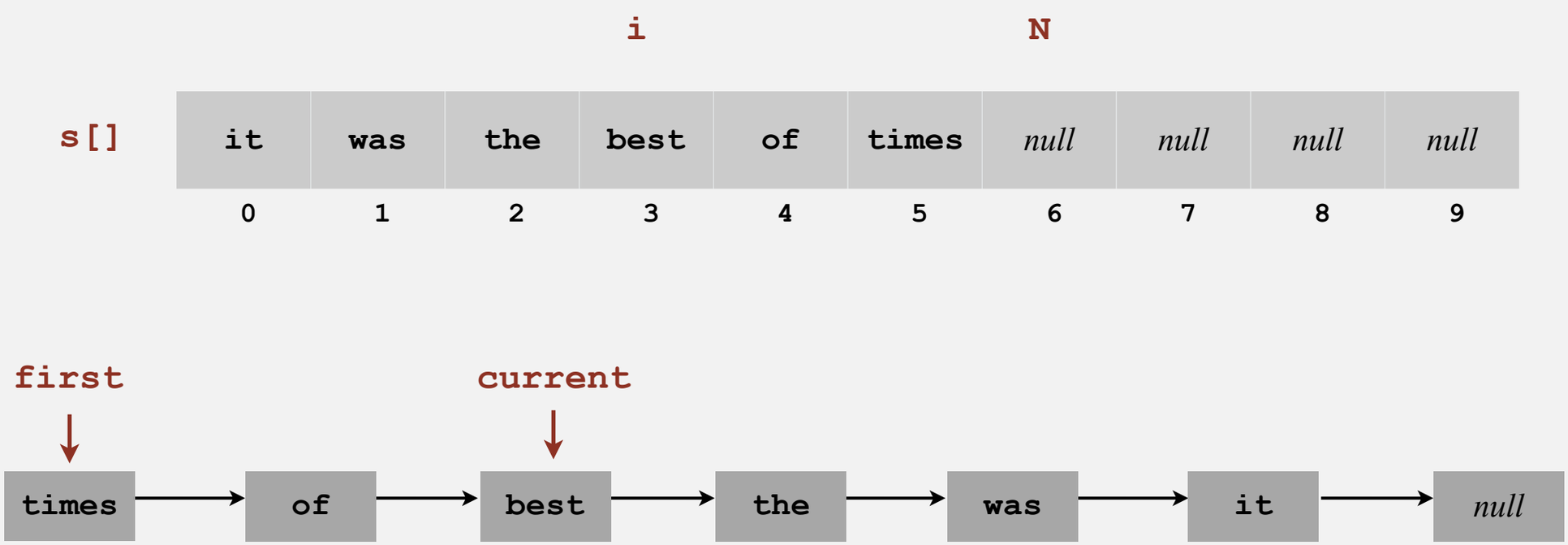
Iteration

Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.



Java solution. Make stack implement the `Iterable` interface.

Iterators

Q. What is an `Iterable` ?

A. Has a method that returns an `Iterator`.

Q. What is an `Iterator` ?

A. Has methods `hasNext()` and `next()`.

Q. Why make data structures `Iterable` ?

A. Java supports elegant client code.

Iterable interface

```
public interface Iterable<Item>
{
   Iterator<Item> iterator();
}
```

Iterator interface

```
public interface Iterator<Item>
{
   boolean hasNext();
   Item next();
   void remove();
}
```

← optional; use at your own risk

"foreach" statement

```
for (String s : stack)
   StdOut.println(s);
```

equivalent code

```
Iterator<String> i = stack.iterator();
while (i.hasNext())
{
   String s = i.next();
   StdOut.println(s);
}
```

Stack iterator: linked-list implementation

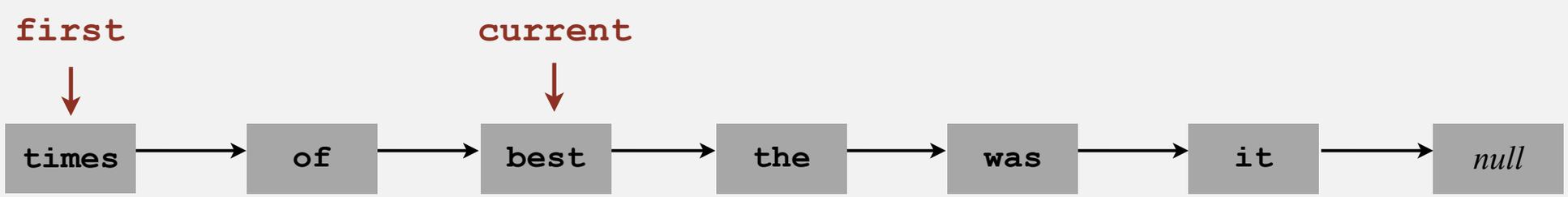
```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

    public Iterator<Item> iterator() { return new ListIterator();  }

    private class ListIterator implements Iterator<Item>
    {
        private Node current = first;

        public boolean hasNext() {  return current != null;  }
        public void remove()     {  /* not supported */      }
        public Item next()
        {
            Item item = current.item;
            current   = current.next;
            return item;
        }
    }
}
```



Stack iterator: array implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    …

    public Iterator<Item> iterator() { return new ArrayIterator(); }

    private class ArrayIterator implements Iterator<Item>
    {
        private int i = N;

        public boolean hasNext() {  return i > 0;          }
        public void remove()     {  /* not supported */   }
        public Item next()       {  return s[--i];         }
    }
}
```

				i			N			
s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

- stacks
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- **APIs and applications**

Java collections library

List interface. `java.util.List` is API for ordered collection of items.

```
public interface List<Item> implements Iterable<Item>
```

	`List()`	*create an empty list*
`boolean`	`isEmpty()`	*is the list empty?*
`int`	`size()`	*number of items*
`void`	`add(Item item)`	*append item to the end*
`Item`	`get(int index)`	*return item at given index*
`Item`	`remove(int index)`	*return and delete item at given index*
`boolean`	`contains(Item item)`	*does the list contain the given item?*
`Iteartor<Item>`	`iterator()`	*iterator over all items in the list*
	`...`	

Implementations. `java.util.ArrayList` uses dynamic array; `java.util.LinkedList` uses linked list.

Linked lists in JDK 5.0

- Class LinkedList<E>: Generic class implementing doubly linked lists.

From Java API documentation

All of the operations perform as could be expected for a doubly-linked list. Operations that index into the list will traverse the list from the beginning or the end, whichever is closer to the specified index.

- Often linked lists are implemented directly in applications for transparency or to allow special features.

Problem session

Consider the following code fragments accessing a list A.

```
1. for (int i=0; i<n; i++) A.add(i); // Add to end

2. Iterator<Integer> it = A.iterator();
   while (it.hasNext()) sum += it.next(); // Iterate

3. for (int i=0; i<n; i++) sum += A.get(i); // Iterate using get()

4. for (int i=0; i<n; i++) A.add(0,i); // Add to start
```

For each of the following implementations of A, how do you predict that the running time will differ (for large n)?

```
List<Integer> A = new LinkedList<Integer>();
List<Integer> A = new ArrayList<Integer>();
```

- Class ArrayList<E>: Generic class implementing unbounded arrays.

From Java API documentation

Each ArrayList instance has a capacity. The capacity is the size of the array used to store the elements in the list. It is always at least as large as the list size. As elements are added to an ArrayList, its capacity grows automatically. The details of the growth policy are not specified beyond the fact that adding an element has constant amortized time cost.

- Interface RandomAccess

From Java API documentation

The best algorithms for manipulating random access lists (such as ArrayList) can produce quadratic behavior when applied to sequential access lists (such as LinkedList). Generic list algorithms are encouraged to check whether the given list is an instanceof this interface

Stack application: Function calls

How a compiler implements a function.

- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.

Note. Can always use an explicit stack to remove recursion.

p = 216, q = 192

gcd (216, 192)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else
}
```

p = 192, q = 24

gcd (192, 24)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else
}
```

p = 24, q = 0

gcd (24, 0)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

Stack application: Arithmetic expression evaluation

Goal. Evaluate infix expressions.

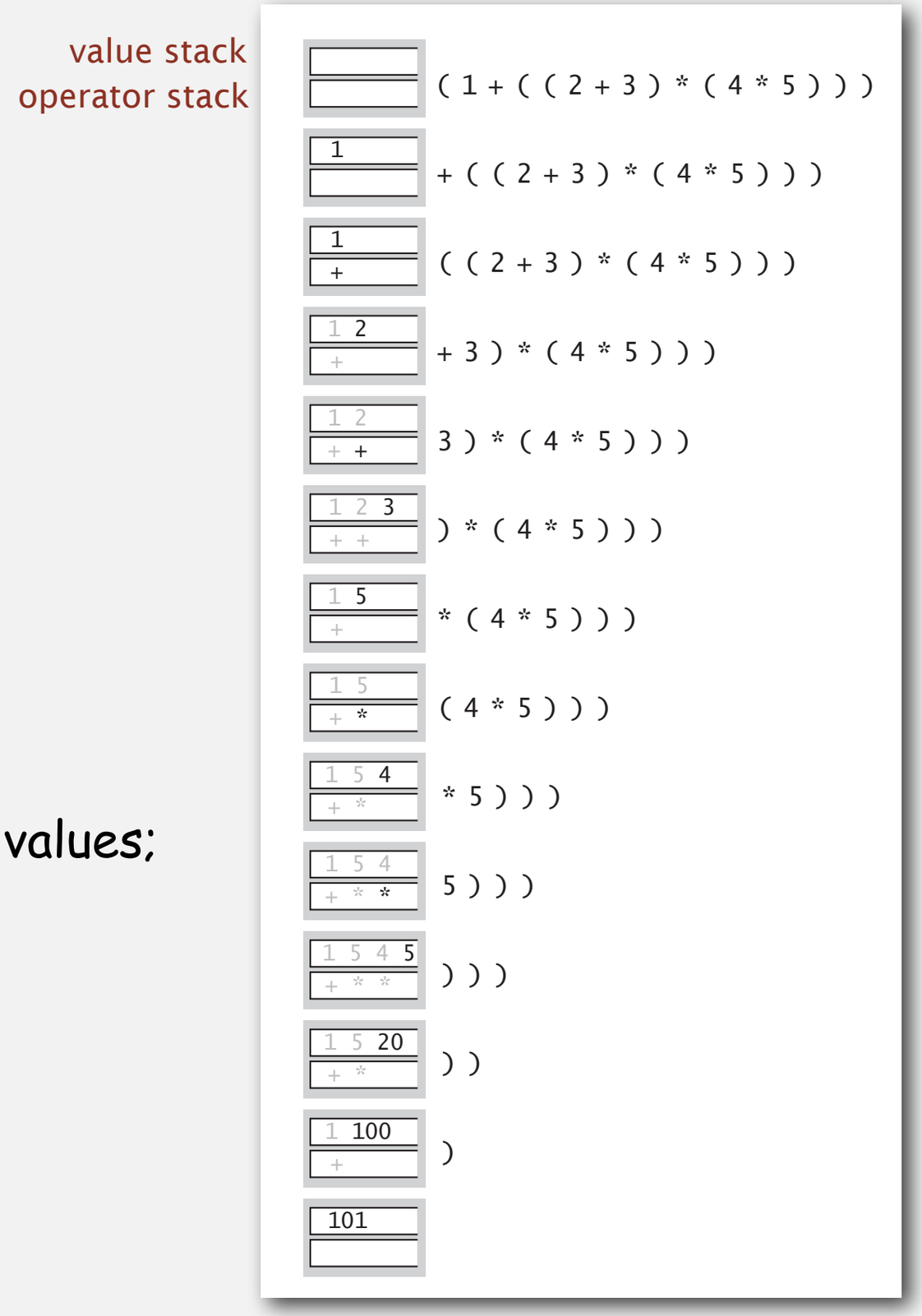
```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

operand operator

Two-stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parenthesis: ignore.
- Right parenthesis: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Context. An interpreter!



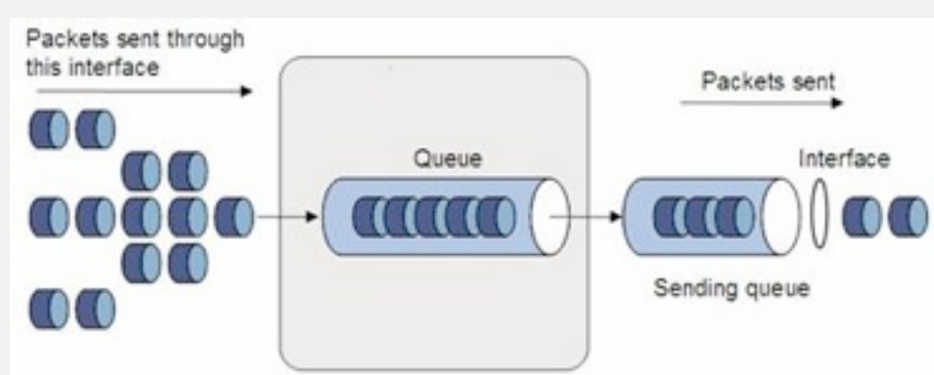
Queue applications

Familiar applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.



Summary of points

- The choice of data structure can have a major impact on the complexity of an algorithm.
- Saw several representations of sequences (one more important representation to come later in course).
- Best choice depends on the usage pattern.
- Amortized analysis is needed to understand the performance of some data structures.
- You are only expected to be able to **use** amortized bounds in algorithm analysis.

Course goals

- The choice of data structure can have a major impact on the complexity of an algorithm.
  - Saw several representations of sequences (one more important representation to come later in course).
  - Best choice depends on the usage pattern.
- Amortized analysis is needed to understand the performance of some data structures.
  - You are only expected to be able to **use** amortized bounds in algorithm analysis, not derive amortized bounds on your own.

Next week

We will next consider fundamental algorithms for sorting.